

$$y = \frac{5x+9}{2x-10}$$

diff. w.r.t. x.

$$\frac{dy}{dx} = \frac{(2x-10)(5) - \cancel{5x+9}(2)}{(2x-10)^2}$$

$$= \frac{(10x-50) - (10x+18)}{(2x-10)^2}$$

$$= \frac{-68}{(2x-10)^2}$$

$$\therefore \frac{dx}{dy} = \frac{-(2x-10)^2}{68}$$

$$\begin{array}{l} 2) \frac{x+2}{x^2+1} \\ 3) \frac{5x+9}{2x-10} \end{array}$$

diff. w.r.t. x .

$$\frac{dy}{dx} = 25 + \frac{2x}{1+x^2} = \frac{25x^2 + 2x + 25}{1+x^2}$$

by derivative of the inverse function

$$\therefore \frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}, \quad \frac{dy}{dx} \neq 0$$

$$\therefore \text{Rate of change of demand with respect to price} = \frac{dx}{dy} = \frac{1+x^2}{25x^2+2x+25}$$

2. Find the marginal demand (x) of a commodity with respect to its price (y) if.

$$1) y = xe^{-x} + 7$$

$$\rightarrow y = xe^{-x} + 7$$

diff. w.r.t. x .

$$\begin{aligned} \frac{dy}{dx} &= x e^{-x} (-1) + e^{-x} \\ &= e^{-x} (1-x) \end{aligned}$$

$$\therefore \frac{dx}{dy} = \frac{1}{e^{-x}(1-x)}$$

$$2) y = \frac{x+2}{x^2+1}$$

$\rightarrow$ diff. w.r.t. x .

$$\therefore \frac{dy}{dx} = \frac{(x^2+1) - (x+2)(2x)}{(x^2+1)^2}$$

$$\frac{dx}{dy} = \frac{(x^2+1)^2}{(x^2+1) - (x+2)(2x)}$$

Exercise 3.2

1. Find the rate of change of demand (x) of a commodity with respect to its price (y)

1) $y = 12 + 10x + 25x^2$

→ Let $y = 12 + 10x + 25x^2$

diff. w.r.t. x .

$$\frac{dy}{dx} = 10 + 50x$$

by derivative of the inverse function

$$\therefore \frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}, \quad \frac{dy}{dx} \neq 0$$

$$\therefore \text{Rate of change of demand with respect to price} = \frac{dx}{dy} = \frac{1}{10 + 50x}$$

2) $y = 18x + \log(x-4)$

→ Let $y = 18x + \log(x-4)$

diff. w.r.t. x .

$$\frac{dy}{dx} = 18 + \frac{1}{x-4} = \frac{18x - 72 + 1}{x-4} = \frac{18x - 71}{x-4}$$

by derivative of the inverse function

$$\therefore \frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}, \quad \frac{dy}{dx} \neq 0$$

$$\therefore \text{rate of change of demand with respect to price} = \frac{dx}{dy} = \frac{x-4}{18x-71}$$

3) $y = 25x + \log(1+x^2)$

→ $y = 25x + \log(1+x^2)$