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Thursday

Heron's Formula

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Date

Exercise 12.1

1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of signal board?

$$\rightarrow 'a' = a, 'b' = a, 'c' = a$$

$$\therefore s = \frac{a+b+c}{2} = \frac{a+a+a}{2} = \frac{3a}{2}$$

$$\therefore \text{Area of signal board} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\# \sqrt{\frac{3a}{2} \left[\frac{3a}{2} \right]}$$

$$= \sqrt{\frac{3a}{2} \left[\frac{3a}{2} - a \right] \left[\frac{3a}{2} - a \right] \left[\frac{3a}{2} - a \right]}$$

$$= \sqrt{\frac{3a}{2} \left[\frac{a}{2} \right] \left[\frac{a}{2} \right] \left[\frac{a}{2} \right]}$$

$$= \sqrt{\frac{3a^4}{16}} = \frac{\sqrt{3}}{4} a^2$$

$$\text{Perimeter} = 180 \text{ cm}$$

$$a+b+c = 180^\circ$$

$$a+a+a = 180^\circ$$

$$3a = 180^\circ$$

$$a = \frac{180^\circ}{3} = 60 \text{ cm}$$

$$\therefore \text{Area of the signal board} \\ = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} (60)^2 = 900\sqrt{3} \text{ cm}^2$$

2. The triangular sides walls of a flyover have been used for advertisements. The sides of the walls are 12m, 22m and 120m. The advertisements yield an earning of ₹5000 per m² per year. A company hired one of its walls for 3 months. How much rent did it pay?

$$\rightarrow a=12\text{m}, b=22\text{m}, c=120\text{m} \\ s = \frac{a+b+c}{2} = \frac{12+22+120}{2} \text{ m} = \frac{154}{2} = 77 \text{ m}$$

$$\therefore \text{Area of wall} = \sqrt{s(s-a)(s-b)(s-c)} \\ = \sqrt{77(77-12)(77-22)(77-120)} \\ = \sqrt{77(65)(55)(-43)} \\ = \sqrt{12 \times 11 \times 10 \times 11 \times 10 \times 12} \\ = 12 \times 11 \times 10 = 1320 \text{ m}^2$$

$$1 \text{ year} = 12 \text{ months}$$

$$\therefore \text{Rent for 12 months per m}^2 = ₹5000$$

$$\therefore \text{Rent for 1 month per m}^2 = \frac{₹5000}{12}$$

$$\therefore \text{Rent for 3 months per m}^2 = \frac{₹5000 \times 3}{12}$$

$$= ₹1250$$

$$\therefore \text{Rent for 3 months of } 1320 \text{ m}^2 \\ = (1250 \times 1320) = ₹1650000.$$

3. There is a slide in a park. One of its side walls has been painted in some colour with a message "Keep the park green and clean". If sides of the wall are 15m, 11m and 6m, find the area painted in colour.

$$\rightarrow a=15\text{m}, b=11\text{m}, c=6\text{m}$$

$$\therefore s = \frac{a+b+c}{2} = \frac{15+11+6}{2} = 16\text{m}$$

$$\therefore \text{Area painted in colour} = \sqrt{s(s-a)(s-b)(s-c)} \\ = \sqrt{16(16-15)(16-11)(16-6)} \\ = \sqrt{16 \times 1 \times 5 \times 10} = 20\sqrt{2} \text{ m}^2.$$

4. Find the area of a triangle two sides of which are 18cm, and 10cm and perimeter is 42m.

$$\rightarrow a=18\text{cm}, b=10\text{cm}$$

$$\text{Perimeter} = 42\text{cm}$$

$$a + b + c = 180^\circ \text{ } 42$$

$$18 + 10 + c = 42$$

$$28 + c = 42$$

$$c = 42 - 28 = 14 \text{ cm}$$

$$s = \frac{a+b+c}{2} = \frac{21}{2} = 21 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{21(21-18)(21-10)(21-14)} \\ &= \sqrt{21 \times 3 \times 11 \times 7} = \sqrt{7 \times 3 \times 3 \times 11 \times 7} \\ &= 7 \times 3 \sqrt{11} = 21\sqrt{11} \text{ cm}^2 \end{aligned}$$

5. Sides of a triangle are in the ratio of 12:17:25 and its perimeter is 540 cm. Find its area.

→ Let the sides of a triangle be $12k$, $17k$ and $25k$.

Then,

$$\text{Perimeter} = 12k + 17k + 25k = 54k \text{ cm.}$$

According to the question,

$$54k = 540$$

$$k = \frac{540}{54} = 10$$

$$\therefore a = 12k = 12 \times 10 = 120 \text{ cm}$$

$$b = 17k = 17 \times 10 = 170 \text{ cm}$$

$$c = 25k = 25 \times 10 = 250 \text{ cm}$$

$$\therefore s = \frac{a+b+c}{2} = \frac{120+170+250}{2} = \frac{540}{2} = 270 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{270(270-120)(270-170)(270-250)} \\ &= \sqrt{270(150)(100)(20)} \\ &= \sqrt{(9 \times 30)(5 \times 30)(5 \times 20)(20)} \\ &= (3)(30)(5)(20) = 9000 \text{ cm}^2 \end{aligned}$$

6. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

$$\rightarrow a=12 \text{ cm}, b=12 \text{ cm}$$

$$\text{Perimeter} = 30 \text{ cm}$$

$$a+b+c=30$$

$$12+12+c=30$$

$$24+c=30$$

$$c=30-24=6 \text{ cm}$$

$$s = \frac{30}{2} = 15 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{15(15-12)(15-12)(15-6)} \\ &= \sqrt{15(3)(3)(9)} \\ &= 9\sqrt{15} \text{ cm}^2 \end{aligned}$$