

Heron's Formula

12

CHAPTER

Tips and Tricks

Area of a rectangle = length \times breadth

Perimeter of a rectangle = $2(\text{length} + \text{breadth})$.

Diagonal of a rectangle = $\sqrt{(\text{length})^2 + (\text{breadth})^2}$

Area of a square = $(\text{side})^2$

Perimeter of a square = $4 \times \text{side}$

Diagonal of a square = $\sqrt{2} \times \text{side}$.

Area of a triangle = $\frac{1}{2}bh$ where b is the base and h is the altitude.

Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ where

$s = \frac{a+b+c}{2}$ is the semi-perimeter of the triangle whose sides are a , b and c . This is called Heron's formula.

Area of an isosceles triangle = $\frac{a}{4}\sqrt{4b^2 - a^2}$ where a is the base and equal sides are b each.

Area of an equilateral triangle of side a
$$= \frac{\sqrt{3}}{4} a^2.$$

Area of a parallelogram = bh where b is the base and h is the altitude.

Area of a rhombus with diagonals d_1 and d_2
$$= \frac{1}{2} d_1 \times d_2.$$

Perimeter of a rhombus with diagonals d_1 and d_2
$$= 2\sqrt{d_1^2 + d_2^2}.$$

- Area of a trapezium = $\frac{1}{2}(a+b)h$ where a and b are the parallel sides and h is the distance between the parallel sides.

- Area of regular hexagon of side a
$$= 6 \times \text{Area of an equilateral triangle of side } a$$

$$= 6 \times \frac{\sqrt{3}}{4} a^2$$

$$= \frac{3\sqrt{3}}{2} a^2.$$

ILLUSTRATIVE EXAMPLES

- An isosceles triangle has perimeter 30 cm and each of its equal side is 12 cm. Find area of the triangle.

Sol. Third side = $30 - (12 + 12) = 6$ cm

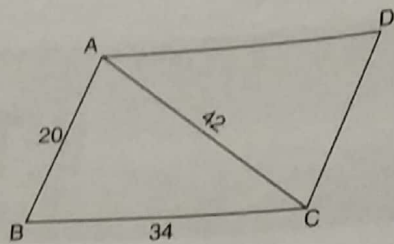
$$\therefore a = 12, b = 12, c = 6$$

$$\therefore s = \frac{a+b+c}{2} = \frac{12+12+6}{2} = 15$$

$$\begin{aligned}\therefore \Delta &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{15(15-12)(15-12)(15-6)} \\ &= \sqrt{15 \times 3 \times 3 \times 9} \\ &= 9\sqrt{15} \text{ cm}^2\end{aligned}$$

2. The adjacent sides of a parallelogram measure 34 cm and 20 cm and the diagonal AC measures 42 cm. Find the area of the parallelogram.

Sol. For $\triangle ABC$



$$a = 34 \text{ cm}$$

$$b = 42 \text{ cm}$$

$$c = 20 \text{ cm}$$

$$\therefore s = \frac{a+b+c}{2} = \frac{34+42+20}{2} = 48 \text{ cm}$$

$$\begin{aligned} \therefore \text{ar}(\triangle ABC) &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{48(48-34)(48-42)(48-20)} \\ &= \sqrt{48 \times 14 \times 6 \times 28} \\ &= 336 \text{ cm}^2 \end{aligned}$$

\therefore area of the parallelogram

$$= 2 \text{ ar}(\triangle ABC)$$

$$= 672 \text{ cm}^2$$

3. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 15 cm, 14 cm and 13 cm and the parallelogram stands on the base 15 cm, find the height of the parallelogram.

Sol. For triangle

$$a = 15 \text{ cm}$$

$$b = 14 \text{ cm}$$

$$c = 13 \text{ cm}$$

$$\therefore s = \frac{a+b+c}{2} = \frac{15+14+13}{2} = 21 \text{ cm}$$

\therefore Area of the triangle

$$\begin{aligned} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{21(21-15)(21-14)(21-13)} \\ &= \sqrt{21 \times 6 \times 7 \times 8} \end{aligned}$$

$$= 84 \text{ cm}^2$$

Let the height of the parallelogram be h cm.
Then, area of the parallelogram

$$= \text{Base} \times \text{Height}$$

$$= 15 \times h \text{ cm}^2$$

According to the question,

$$15h = 84$$

$$\Rightarrow h = \frac{84}{15} = 5.6 \text{ cm}$$

Hence, the height of the parallelogram is 5.6 cm

4. A triangular park in a city has dimensions 30 m \times 26 m \times 28 m. A gardener has to plant grass inside the park at ₹1.50 per m^2 . Find the amount to be paid to the gardener.

Sol. For triangular park

$$a = 30 \text{ m}$$

$$b = 26 \text{ m}$$

$$c = 28 \text{ m}$$

$$\therefore s = \frac{a+b+c}{2} = \frac{30+26+28}{2} = 42 \text{ m}$$

\therefore Area of the triangular park

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{42(42-30)(42-26)(42-28)}$$

$$= \sqrt{42 \times 12 \times 16 \times 14}$$

$$= 336 \text{ m}^2$$

\therefore Amount to be paid to the gardener

$$= 336 \times 1.50 = ₹ 504$$

5. The perimeter of a triangular ground is 900 and its sides are in the ratio 3 : 5 : 4. Using Heron's formula, find the area of the ground.

Sol. Perimeter = 900

$$\text{Ratio} = 3 : 5 : 4$$

$$\text{Sum of ratios} = 3 + 5 + 4 = 12$$

$$\therefore \text{Sides are } \frac{3}{12} \times 900, \frac{5}{12} \times 900, \frac{4}{12} \times 900$$

$$\text{or } 225, 375, 300$$

$$\therefore a = 225$$

$$b = 375$$

$$c = 300$$

$$s = \frac{a + b + c}{2} = \frac{225 + 375 + 300}{2} = 450$$

Area of the triangular ground

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{450(450-225)(450-375)(450-300)}$$

$$= \sqrt{450 \times 225 \times 75 \times 150}$$

$$= 33750$$

The base of an isosceles triangle measures 24 cm and its area is 60 m^2 . Find its perimeter.

$$\text{Area} = \frac{a}{4} \sqrt{4b^2 - a^2}$$

$$\Rightarrow 60 = \frac{24}{4} \sqrt{4b^2 - (24)^2}$$

$$\Rightarrow 10 = \sqrt{4b^2 - 576}$$

$$\Rightarrow 4b^2 - 576 = 100 \quad | \text{ Squaring both sides}$$

$$\Rightarrow 4b^2 = 676$$

$$\Rightarrow b^2 = 169$$

\Rightarrow

$$b = 13 \text{ m}$$

$$\therefore \text{Perimeter} = a + b + c$$

$$= 24 + 13 + 13$$

$$= 50 \text{ m}$$

7. The unequal side of an isosceles triangle is 6 cm and its perimeter is 24 cm. Find its area.

Sol. $b = 6 \text{ cm}$

$$a + a + b = 24$$

$$\Rightarrow 2a + 6 = 24$$

$$\Rightarrow a = 9 \text{ cm}$$

$$\therefore \text{Area} = \frac{a}{4} \sqrt{4b^2 - a^2}$$

$$= \frac{9}{4} \sqrt{4(6)^2 - (9)^2}$$

$$= \frac{9}{4} \sqrt{144 - 81}$$

$$= \frac{9}{4} \sqrt{63}$$

$$= \frac{9}{4} 3\sqrt{7} \text{ cm}^2$$

Internal Assessment

ORAL QUESTIONS

What is the perimeter of a triangle?

What is the semi-perimeter of a triangle?

How many sides of an isosceles triangle are equal?

What is the angle between the diagonals of a rhombus?

What is the perimeter of a regular hexagon of side a ?

TRUE OR FALSE

Area of a rectangle is equal to $2(\text{length} \times \text{breadth})$.

Diagonal of a rectangle = $\sqrt{(\text{length})^2 + (\text{breadth})^2}$.

Semi-perimeter of a triangle with sides a , b and c is equal to $\frac{a+b+c}{2}$.

Area of a triangle with sides a , b and c is equal to $\sqrt{s(s-a)(s-b)(s-c)}$ where $2s = a + b + c$.

5. Area of an isosceles triangle is equal to $\frac{a}{4} \sqrt{4b^2 - a^2}$ where a is the base and b is each of the equal sides.

6. The area of the equilateral triangle is $9\sqrt{3} \text{ cm}^2$ where each side is 6 cm.

7. The area of a regular hexagon of side ' a ' is the sum of the areas of eight equilateral triangles with side a .

8. The base and the corresponding altitude of a parallelogram are 8 cm and 3 cm respectively. The area of the parallelogram is 12 cm^2 .

9. The area of the isosceles triangle is $\frac{5}{4} \sqrt{39} \text{ cm}^2$, if the perimeter is 13 cm and the base is 5 cm.

10. In a triangle, the sides are given as 11 cm, 12 cm and 13 cm. The length of the altitude is $\frac{12\sqrt{105}}{11} \text{ cm}$ corresponding to the side having length 11 cm.

Assignments

Name:	Class:	Section:	Roll No.:	Grade:	Teacher's sign:
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CLASS ASSIGNMENT 1

- What is the perimeter of a triangle whose sides are 6 cm, 7 cm and 8 cm?

- What is the area of a triangle whose base is 5 cm and altitude is 4 cm?

- What is the area of a rectangle whose length is 4 cm and breadth is 2.5 cm.

- What is the perimeter of a rectangle whose length is 6 cm and breadth is 3.5 cm?

- What is the semi-perimeter of a triangle whose sides are 4 cm, 5 cm and 6 cm?

- What is the length of the diagonal of a rectangle whose length is 40 cm and breadth is 30 cm?

- What is the perimeter of a square of side 3 cm?

- What is the area of a square of side 2 cm?

- What is the diagonal of a square of side 4 cm?

- What is the area of an isosceles triangle whose base is 5 cm and equal sides are 4 cm each?

