Polynomials

Division Algorithm

If $p(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$, then –
$p(x) = g(x) \times q(x) + r(x)$
where, $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$

Relationship-Zeroes and Coefficient of Polynomials

Quadratic

α and β are zeroes of Quadratic Polynomial

$ax^2 + bx + c$

Then, Sum of zeroes,

$\alpha + \beta = -\dfrac{b}{a}$

Product of zeroes

$\alpha\beta = \dfrac{c}{a}$

Cubic

α, β and γ are zeroes of Cubic Polynomial

$ax^3 + bx^2 + cx + d$

Sum of zeroes,

$\alpha + \beta + \gamma = -\dfrac{b}{a}$

Sum of products of the zeroes taken two at a time

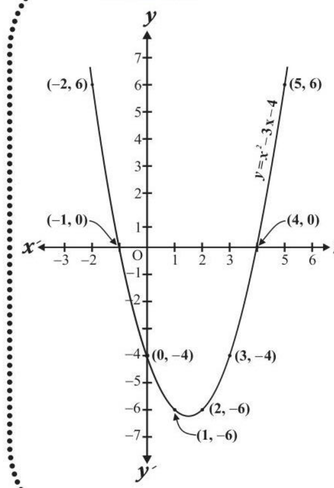
$\alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{c}{a}$

Product of zeroes

$\alpha\beta\gamma = -\dfrac{d}{a}$

Graphical Representation Quadratic Polynomial

Parabola



$y = x^2 - 3x - 4$

Degree of Polynomial

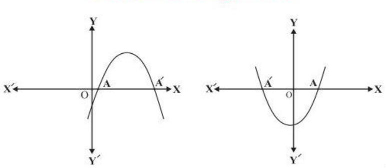
Highest power of x in Polynomial, $p(x)$

Types

Polynomial	Degree	General Form
Linear	1	$ax+b$
Quadratic	2	ax^2+bx+c, $a \neq 0$
Cubic	3	ax^3+bx^2+cx+d, $a \neq 0$

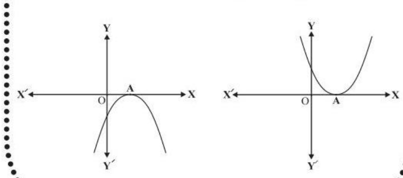
Zeroes of Polynomial Graphically

Case1- Graph cuts x-axis at 2 points



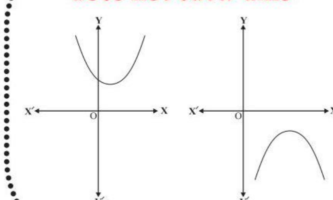
Number of Zeroes 2

Case2- Graph cuts x-axis at exactly one point



Number of Zeroes 1

Case3- Graph does not cut x-axis



Number of Zeroes 0

POLYNOMIALS

Let x be a variable, n be a +ve integer and $a_1, a_2, a_3 \ldots a_n$ be constants (real numbers) then,

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots a_1 x + a_0$$

is called a polynomial in variable x.

In the polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots a_0$

$a_n x^n, a_{n-1} x^{n-1} \ldots a_1 x$ & a_0 are terms of the polynomial and $a_n, a_{n-1}, a_{n-2} \ldots a_1$ and a_0 are their coefficients.

Ex: $p(x) = 3x - 2$ is polynomial in variable x
$q(y) = 3y^2 - 2y + 4$ is a polynomial in variable y

DEGREE OF A POLYNOMIAL

The highest exponents in a polynomial, is called its degree:

Ex: $f(x) = 5x + \frac{1}{3}$ is a polynomial of degree I

$g(x) = 3y^2 - \frac{5}{2}y + 7$ is a polynomial of degree 2

TYPES OF A POLYNOMIALS

1) Zero polynomial :- the degree of the zero polynomial is not defined,

Ex: $f(x)=0$, $g(x)=0.x^2$, $p(x)=0.x^3$

2) Constant polynomial :- A polynomial of degree zero is called a constant polynom-ial.

Ex: $f(x)=7$, $g(x)=8$, $p(x)=-\frac{3}{2}$

3) Linear polynomial :- A polynomial of degree 1 is called a linear polynomial

Ex: $f(x)=4x-3$, $g(x)=\sqrt{3}t-5$

$f(x)=ax+b$, $a\neq 0$

4) Quadratic polynomial :- A polynomial of degree 2 is called a quadratic polyno-mial

Ex: $f(x)=ax^2+bx+c$, $a\neq 0$,

$p(x)=2x^2+3x$

$g(u)=2-u^2+\sqrt{3}u$

5) cubic polynomials :- A polynomial of degree 3 is called cubic polynomial.

~~Ex:~~ $\cancel{ax^3}$ $f(x)=$ $\cancel{a^3+b}$

Ex: $f(x) = ax^3 + bx^2 + cx + d$; $a \neq 0$

$p(x) = 2x^3 + 5x - 7$

6) **Bi-quadratic polynomial**: A fourth degree polynomial is called a biquadratic polynomial.

Ex: $f(x) = ax^4 + bx^3 + cx^2 + dx + e$, $a \neq 0$

$p(y) = 2y^4 + 3$

Value of a polynomials :-

If $f(x)$ is a polynomial and α is any real number, then the real number obtained by replacing x by α in $f(x)$, is called the value of $f(x)$ at $x = \alpha$ and is denoted by $f(\alpha)$.

Examples :-

1) Find the value of $f(x) = 2x^2 - 3x - 2$ at $x = 1$

$f(x) = 2x^2 - 3x - 2$, $x = 1$

$f(1) = 2(1)^2 - 3(1) - 2$

$= 2 \times 1 - 3 - 2$

$= 2 - 3 - 2$

$f(1) = -3$

Thus the value of $f(x)$ at $x = 1$ is -3

2] Find the value of $p(x) = 5x^2 - 4x + 2$ at $x = 2$.

$p(x) = 5x^2 - 4x + 2$, $x = 2$

$p(2) = 5(2)^2 - 4(2) + 2$

$p(2) = 5 \times 4 - 8 + 2$

$= 20 - 8 + 2$

$= 12 + 2$

$p(2) = 14$

Thus the value of $p(x)$ at $x = 2$ is 14

ZERO OF A POLYNOMIALS :-

A real number α is said to be a zero of polynomial $f(x)$, if $f(\alpha) = 0$

Examples

1) If $p(x) = 5x + 7$, then find the zero of the polynomial.

$p(x) = 5x + 7$

put $p(x) = 0$

$0 = 5x + 7$

$5x = -7$

$x = \frac{-7}{5}$

$\therefore$ Zero of $p(x)$ is $\frac{-7}{5}$

2) If $f(x) = ax + b$, find the zero of the polynomial.

$$f(x) = ax + b$$
$$0 = ax + b$$
$$ax = -b$$
$$x = \frac{-b}{a}$$

~~3) Find the value of 'a'~~

3) If 2 is a zero of a polynomial $p(x) = 4x^2 + 2x - 5a$, then find the value of a.

$\Rightarrow$ $p(x) = 4x^2 + 2x - 5a$

Since 2 is zero of $p(x)$

$\Rightarrow$ $p(2) = 0$

$$4(2)^2 + 2(2) - 5a = 0$$
$$4 \times 4 + 4 - 5a = 0$$
$$16 + 4 - 5a = 0$$
$$20 - 5a = 0$$
$$5a = 20$$
$$a = \frac{20}{5}$$
$$\boxed{a = 4}$$

4) Find the value of a if $x+a$ is a factor of the polynomial $2x^2+2ax+5x+10$.

$\Rightarrow$ Let $p(x) = 2x^2+2ax+5x+10$

Since $x+a$ is factor of $p(x)$.

$\Rightarrow p(-a) = 0$

$2(-a)^2+2a(-a)+5(-a)+10 = 0$

$\cancel{2a^2} - \cancel{2a^2} - 5a + 10 = 0$

$5a = 10$

$a = \frac{10}{5}$

$\boxed{a=2}$

Geometrical meaning of the zeroes of a polynomial

If we plot the graph of $y = p(x)$ on a graph paper then α is a zero of $p(x)$ if and only if $(\alpha, 0)$ lies on the graph of $y = p(x)$

Examples: 1) Let $p(x) = 2x+3$ then $y = p(x)$ meets the x-axis in $(\frac{-3}{2}, 0)$ note that $p(\frac{-3}{2}) = 0$.

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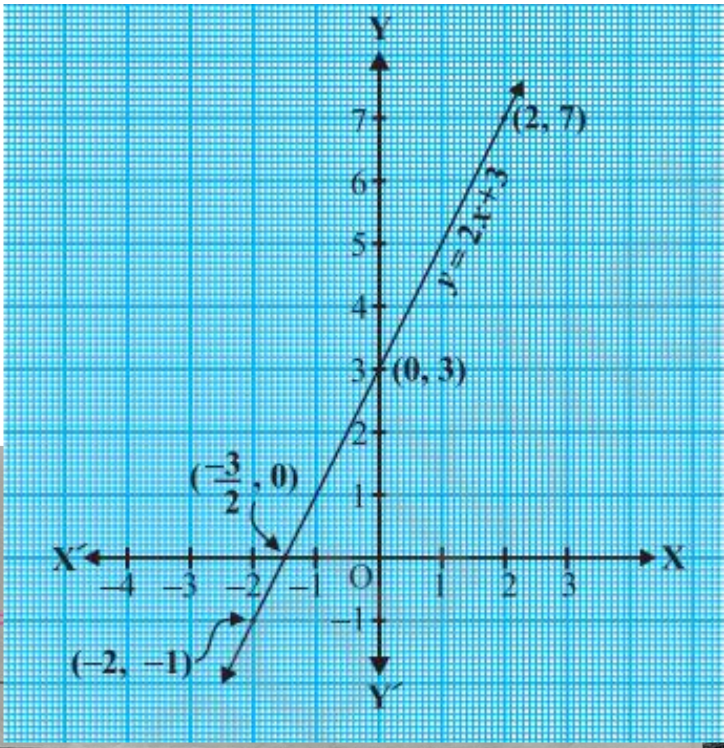

Example 2) Let $p(x) = x^2 - 3x - 4$, then $y = p(x)$ meets the x-axis in $(-1, 0)$ and $(4, 0)$ note that $p(-1) = 0$ and $p(4) = 0$