

Class 9th Sub - Mathematics Writer - Agarwal
chapter = 14 Arc properties of circles

Ans 1

Given that

$$\text{Arc AC} = \text{Arc BD}$$

To prove $\Rightarrow AB \parallel CD$

Now,

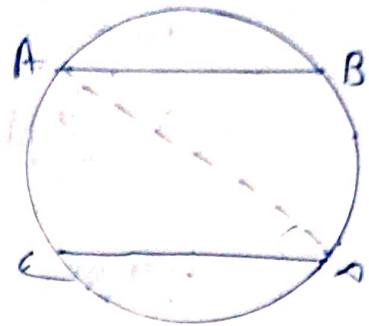
$$\text{proof} = \text{Arc AC} = \text{Arc BD}$$

$\therefore \angle ADC = \angle BAD$ (Arc are equal so its angles also will be equal)

But, these are alternate angles

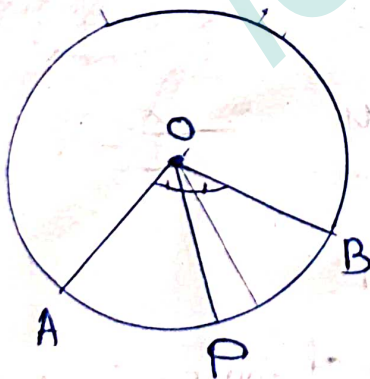
Therefore $AB \parallel CD$

Proved.



Ans 2. Given that an arc AB

which subtends $\angle AOB$ at centre O is the mid point of AB



Join OP

To prove $\angle AOP = \angle BOP$

Proof: $AC = BC$ (P is the mid point of Arc AB)

these subtend $\angle AOP$ and $\angle BOP$

$\therefore \angle AOP = \angle BOP$ (OP is bisector)

Hence proved.

3 Another given that

P is mid point of arc APB

M is mid point of chord AB

A

Now,

Join AO and BO and produce OM at Q .

(i) $PM \perp AB$

Proof: - Arc $AP = BP$

Therefore $\angle AOP = \angle POB$

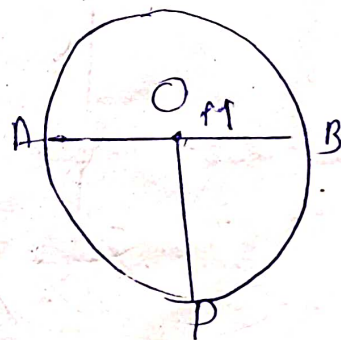
$\angle AOM = \angle BOM$

Now, $\triangle OAM$ and $\triangle OBM$

$OB = OA$ (radius of same circle)

$OM = OM$ (common)

$\angle AOM = \angle BOM$ (prove already)



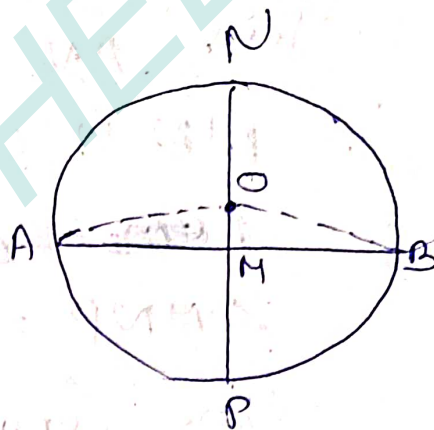
Therefore $\angle BMO = \angle AMO$

As,

$$\angle AMO + \angle BMO = 180^\circ$$

$$\angle AMO = \angle BMO = 90^\circ$$

Hence $MP \perp AB$ or OM



$$(i) \angle AMP + \angle OMA = 90^\circ + 90^\circ \\ \Rightarrow 180^\circ$$

PMO is a straight line. passes through O

(ii) since PO is diameter of circle

$$\text{arc } PAQ = \text{arc } PBQ$$

$$\text{arc } PAQ - \text{arc } AP = \text{arc } PBQ - \text{arc } PB$$

$$(AQ = PB)$$

$$\text{arc } AN = BN$$

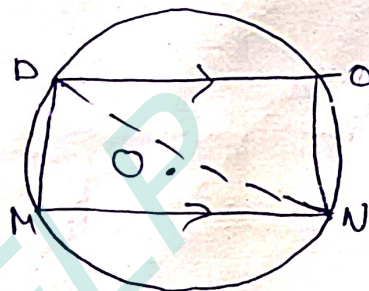
N is the bisector of AB

Proved

Answer Given that $MNOP$ is a cyclic trapezium

$$MN \parallel OP$$

$$\uparrow \text{ to prove } = MP = NO$$



Proof = Join PN

$\therefore MN \parallel OP$ (given)

$$\angle \text{---} = \angle 1$$

$$\angle MNP = \angle OPN \text{ (alternate angles)}$$

these angles subtend by the

Arcs MP and NO

$$\text{arc MP} = \text{arc NO}$$

$$\text{chord MP} = \text{chord NO}$$

[equal arc = equal chord]

Hence $MP = NO$

Proved.

5

A circle has O centre and P is point.

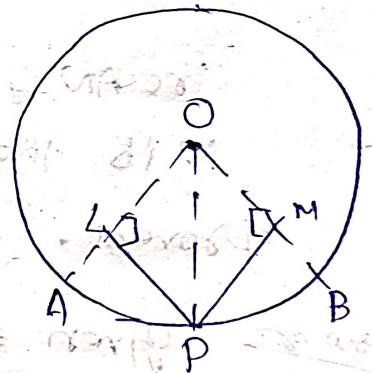
radii is OA and OB

To prove = arc AP = arc PB

Now Join OP

Proof:- in right angled $\triangle OMP$ and $\triangle OBP$

$$OP = OP \text{ (common)}$$



$$PL = PM \text{ (given)}$$

$$\angle LOP = \angle MOP \text{ (given by CPCT)}$$

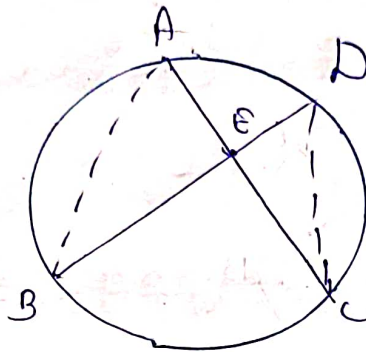
$$\therefore \triangle OLP \cong \triangle OMP \text{ (RHS)}$$

$$\therefore \angle AOP = \angle BOP$$

$$\therefore \text{Arc AP} = \text{Arc BP} \text{ (Because angles are equal so arcs will also)}$$

Proved.

6 Answer



Given that

in a figure, there is two chords AC and BD both intersect to each other at a point E.

$$\text{Arc AB} = \text{Arc CD}$$

To prove:- $AE = ED$

$$BE = EC$$

construction = Join CD and AD

proof:

$$\text{Chord AB} = \text{Chord CD} \text{ (given)}$$

$$\text{Arc AB} = \text{Arc CD} \text{ (given)}$$

Now in $\triangle CED$ and $\triangle BEA$

$$CD = AB \text{ (proved)}$$

$$\angle FAB = \angle EDC \text{ (angle in same segment)}$$

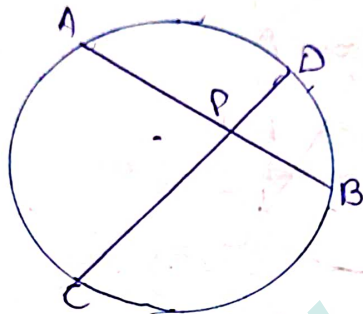
$$\angle FBA = \angle DCE \text{ (angle in same segment)}$$

$$\therefore \triangle BEA \cong \triangle CED \text{ (A.S.A)}$$

$$BE = EC \text{ (c.p.c.t)}$$

$$AE = DE \text{ (c.p.c.t)}$$

Answer
7th



Given that two chords AB and CD intersect each other at point P
So,

$$AB = CD$$

$$\text{To prove} = \text{Arc AD} = \text{Arc CB}$$

$$\text{Proof} = AB = CD \text{ (given)}$$

$$\text{Arc AB} = \text{Arc CD}$$

Subtract arc BD from both sides,

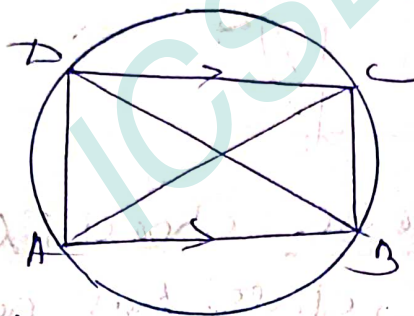
$$\text{Arc AB} - \text{arc BD} = \text{Arc CD} - \text{arc BD}$$

$$\Rightarrow \text{Arc AD} = \text{Arc CB}$$

proved

Answer 8

Given that a cyclic quadrilateral $ABCD$
in it AC and BD are diagonal
 $AB \parallel DC$



To prove:- (i) $AD = BC$
(ii) $AC = BD$

Proof = in $ABCD$ quadrilateral

$AB \parallel DC$ (given)

$\therefore \angle BAC = \angle ACD$ (alternate angle)

arc $BC =$ arc AD

(Because of equal angle)

$AD = BC$

(equal arc have equal chords)

Now

in $\triangle ABC$ and $\triangle ADB$

$AB = AB$ (common)

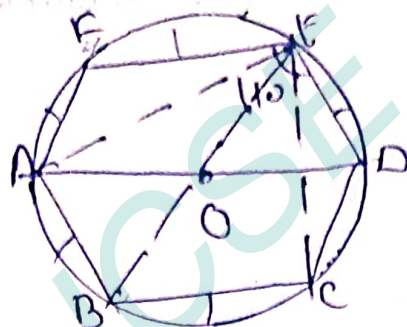
$BC = AD$ (proved)

$\angle ACB = \angle ADB$ (angle in same)

$\triangle ABC \cong \triangle ADB$ (by S.A.S)

$AC = BD$ (by c.p.c.t)

Answer 9th



chord $AB = \text{chord } BC = \text{chord } CD$ (given)

AD is diameter, circle has centre 'O' (given)

$\angle DEF = 110^\circ$ (given)

(i) $\angle AEF = ?$, (ii) $\angle FAB = ?$

Construction: Joining CE , BE and AE

Proof: -

$AB = CD = BC$

$\text{arc } AB = \text{arc } CD = \text{arc } BC$

$\angle AEB = \angle CED = \angle BEC$

(equal arcs subtend equal angles)

and $\angle AED = 90^\circ$ (angle in semi circle)

(i) $\therefore \angle AEF = \angle DEF - \angle AED$
 $= 110^\circ - 90^\circ$

$= 20^\circ$

(ii) Since $ABEF$ is a cyclic quadrilateral

$\angle FAB + \angle BEF = 180^\circ$

$\angle FAB + 50 = 180^\circ$ ($\angle BEF = 30 + 20 = 50$)

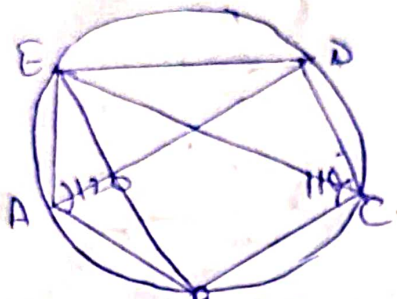
$$\angle FAB = 180^\circ - 50^\circ \\ = 130^\circ \text{ Ans}$$

Answer 10 ABCDEA is a pentagon which is inscribed in a circle.

$$AB = BC = CD \text{ (given)}$$

$$\angle BCD = 110^\circ \text{ (given)}$$

$$\angle BAE = 120^\circ \text{ (given)}$$



Construction = join BE, AD and CE

(i) in cyclic quadrilateral EBCD

$$\angle BCD = 110^\circ$$

$$\angle BED = 180^\circ - 110^\circ \\ \Rightarrow 70^\circ$$

$$BC = AB = CD$$

$$\therefore \text{arc } BC = \text{arc } AB = \text{arc } CD$$

$$\angle CED = \angle BEC$$

$$= \frac{70^\circ}{2} = 35^\circ$$

$$\text{and } \angle AEB = 35^\circ \text{ (Bcoz } AB = CD = BC)$$

(ii) in quadrilateral AECB

$$\angle ABC + \angle AEC = 180^\circ$$

$$\angle ABC + 70^\circ = 180^\circ$$

$$\angle ABC = 180 - 70 \\ = 110^\circ$$

in $\triangle ABE$

$$\angle EAB + \angle ABE + \angle AEB = 180^\circ$$

$$120 + \angle ABE + 35^\circ = 180^\circ$$

$$\angle ABE + 155^\circ = 180^\circ$$

$$\angle ABE = 180 - 155 \\ = 25^\circ$$

$$\therefore \angle EBC = \angle ABC - \angle ABE \\ = 110^\circ - 25^\circ \\ = 85^\circ$$

Now, in quadrilateral EBCD

$$\angle CED + \angle EBC = 180^\circ$$

$$\angle CED + 85^\circ = 180^\circ$$

$$\angle CED = 180 - 85 \\ = 95^\circ$$

$$(11) \angle AED = 35^\circ + 35^\circ + 35^\circ \\ = 105^\circ$$

(iv) in ABCD quadrilateral

$$\angle BCD + \angle DAB = 180^\circ$$

$$110^\circ + \angle DAB = 180^\circ$$

$$\begin{aligned}\angle DAB &= 180 - 110 \\ &= 70^\circ\end{aligned}$$

But

$$\angle EAB = 120^\circ$$

$$\begin{aligned}\angle EAD &= 120 - 70 \\ &= 50^\circ\end{aligned}$$