19

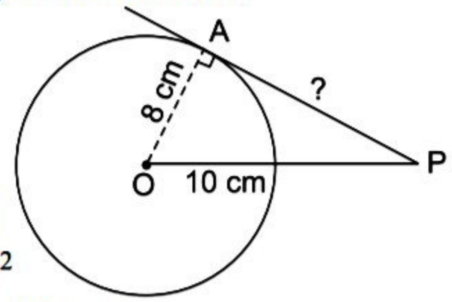
TANGENT PROPERTIES OF CIRCLES

EXERCISE 19A

1. In the circle, OA is radius and AP is the tangent to the circle.

$\therefore$ $OA = 8$ cm, $OP = 10$

$OA \perp AP$. So, $\angle OAP = 90°$



In right ΔOAP, we have

$OP^2 = OA^2 + AP^2$ [Pythagoras theorem]

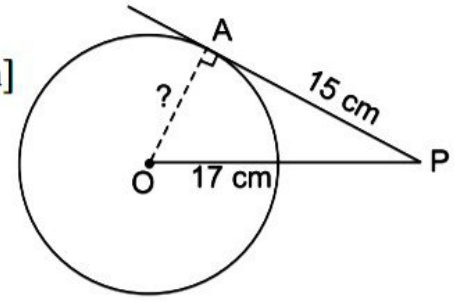
$\Rightarrow (10)^2 = (8)^2 + AP^2 \Rightarrow 100 = 64 + AP^2$

$\Rightarrow AP^2 = 100 - 64 = 36 = (6)^2 \Rightarrow AP = 6$ cm

2. In the circle, OA is radius and AP is the tangent drawn from P.

$\therefore \angle OAP = 90°$. So, $OA \perp AP$

Now, in right ΔOAP, we have



$OP^2 = OA^2 + AP^2$ [Pythagoras Theorem]

$\Rightarrow (17)^2 = OA^2 + (15)^2$

$\Rightarrow 289 = OA^2 + 225$

$\Rightarrow OA^2 = 289 - 225 = 64 = (8)^2$

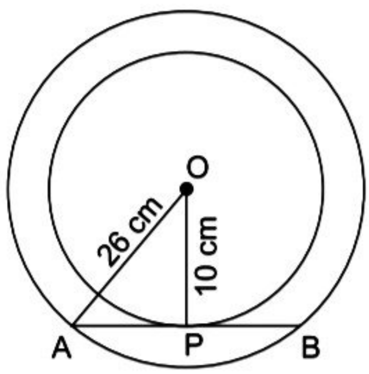
$\therefore OA = 8$

Hence radius of the circle = 8 cm.

3. Radius (r) of the inner circle = 10 cm.

Radius (R) of the outer circle = 26 cm.

AB is the chord of the outer circle and tangent to the inner circle at P.



Join OA and OP.

$\because$ AB is tangent and OP the radius of the inner circle.

$\because$ OP $\perp$ AB and P bisects the chord AB of the outer circle .

Now, in right ΔOAP, we have

$OA^2 = AP^2 + OP^2$ [Pythagoras Theorem]

$\Rightarrow AP^2 = 676 - 100 = 576 = (24)^2$

$\therefore AP = 24$ cm

Henc, $AB = 2AP = 2 \times 24 = 48$ cm.

4. A,B and C are the centres of the three circles , such that circle with centre C touches the other two circles externally.

Radius of circle with centre $A = 9$ cm.

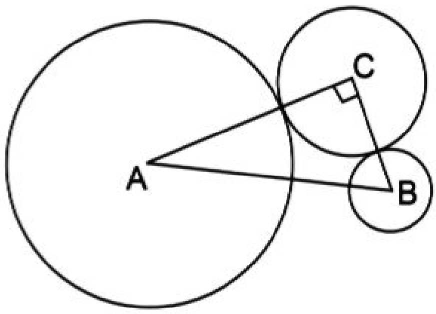
Radius of circle with centre $B = 2$ cm.

$AB = 17$ cm, and $\angle ACB = 90^\circ$

Let radius of the third circle $= r$

$\therefore AC = (9 + r)$ cm and $BC = (2 + r)$cm.

Now, in right ΔACB, we have



$AB^2 = AC^2 + BC^2$

$\Rightarrow (17) 2 = (9 + r)2 + (2 + r)2$

$\Rightarrow 289 = 81 + 18r + r2 + 4 + 4r + r2$

$\Rightarrow 289 = 2r2 + 22r + 85$

$\Rightarrow 2r2 + 22r + 85 - 289 = 0$

$\Rightarrow 2r2 + 22r - 204 = 0$

$\Rightarrow r2 + 11r - 102 = 0$ [Dividing by 2]

Now, $r2 + 17r - 6r - 102 = 0$

$\Rightarrow r (r + 17) - 6 (r + 17) = 0$

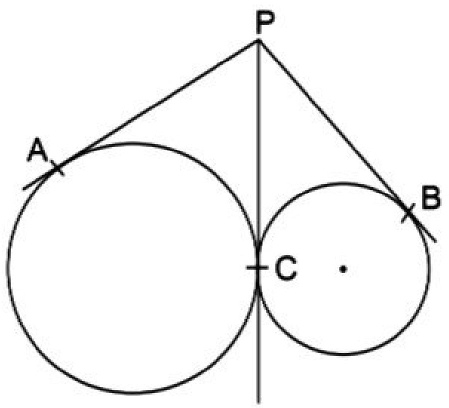
$\Rightarrow (r + 17) (r - 6) = 0$ [Zero product rule]

Either $r + 17 = 0$ then $r = - 17$, but it is not admissible.

Or $r - 6 = 0$, then $r = 6$

Hence, radius of the third circle $(r) = 6$ cm

5. Given: Two circles touch each other externally at C. Through C, a common tangent is drawn. From a point P on it, tangents PA and PB are drawn to their respective circles.



To prove. $PA = PB$

Proof: From P, PA and PC are the tangents drawn to the first circle

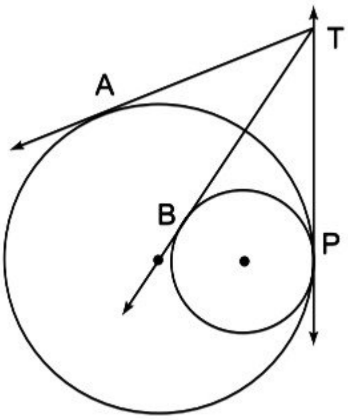
$\therefore PA = PC$ ...(i)

Similarly, from P, PB and PC are the tangents drawn to the second circle.

$\therefore PB = PC$ ...(ii)

From (*i*) and (*ii*) we have, $PA = PB$. **Hence proved**

6. **Given:** Two circles touch each other at P internally. *A* common tangent is drawn from *P*. From a point *T* on it, *TA* and *TB* tangents are drawn to the given two circles.



To prove. $TA = TB$

Proof. $\because$ From T, TA and TP are the tangents to the first circle .

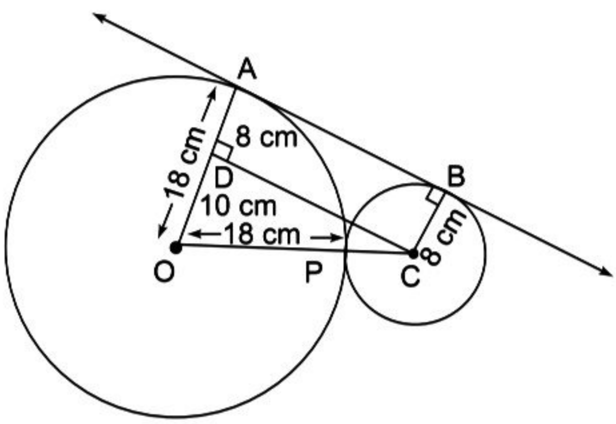
$\therefore TA = TP$ ...(i)

Similarly, From T, TB and TP are the tangents to the second circle

$\therefore TB = TP$ ...(ii)

From (*i*) and (*ii*), we have $TA = TB$. **Hence proved.**

7. Two circles with centre O and C touch each other externally at P.



Radius of the first circle is 18 cm and second circle is 8 cm

AB is the direct common tangent. From C, draw $CD \perp AO$ meeting OA at D.

$\therefore OD = OA - AD = 18 - 8 = 10$ cm.

$$OC = OP + PC = 18 + 8 = 26 \text{ cm.}$$

Now, in right ΔODC, we have

$$OC^2 = OD^2 + DC^2 \qquad \text{[Pythagoras Theorem]}$$

$\Rightarrow (26)^2 = (10)^2 + DC^2 \quad \Rightarrow \quad 676 = 100 + DC^2$

$\Rightarrow DC^2 = 676 - 100 = 576 = (24)^2$

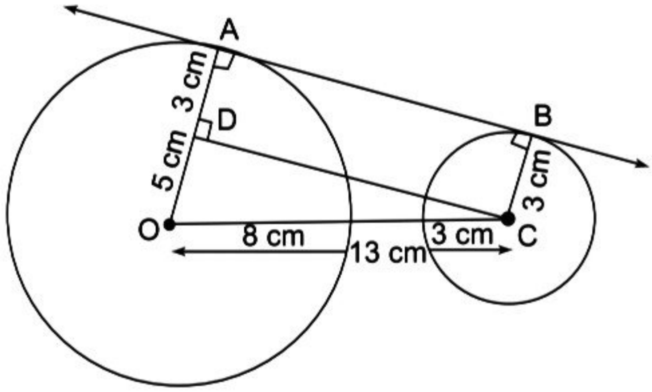
$\therefore DC = 24$ cm

$\therefore$ AB = DC = 24 cm.

8. Two circles with centre *O* and C are drawn of the radii 8 cm and 3 cm. Their centres are 13 cm apart.

AB is their common direct tangent. Join *OA* and *CB*.

Through *C*, draw a perpendicular *CD* to OA meeting it at *D*.



Now, $OD = 8 - 3 = 5$ cm, OC = 13 cm

Now, in right ΔODC, we have

$$OC^2 = OD^2 + DC^2 \qquad \text{[Pythagoras Theorem]}$$

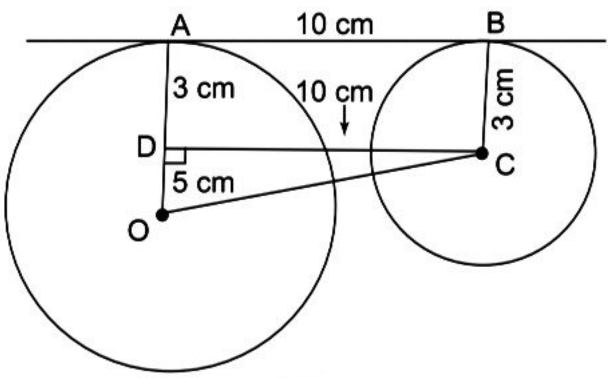
$\Rightarrow (13)^2 = (5)^2 + DC^2$

$\Rightarrow 169 = 25 + DC^2 \quad \Rightarrow \quad DC^2 = 169 - 25 = 144$

$\therefore DC = \sqrt{144} = 12$ cm. $\quad \therefore AB = 12$ cm $\qquad [\because AB = DC]$

9. Two circles of radii 8 cm and 3 cm have *O* and *C* as their centres respectively. AB is their common direct tangent.

OA = 8 cm, CB = 3 cm, *AB* = 10 cm.



$\therefore \quad OD = 8 \text{ cm} - 3 \text{ cm} = 5 \text{ cm}$ and $CD = AB = 10$ cm.

Now, in right ΔDOC, $OC^2 = OD^2 + DC^2$ [Pythagoras Theorem]

$(5)^2 + (10)^2 = 25 + 100 = 125 = 25 \times 5$

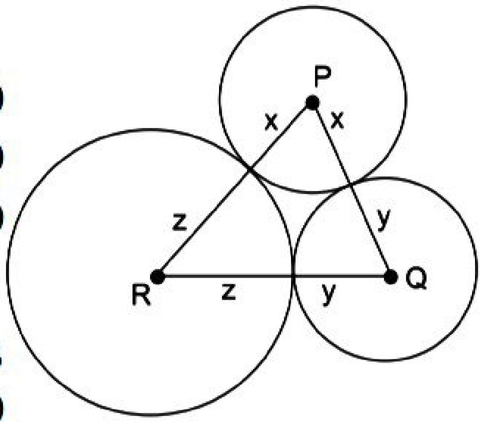
$\therefore OC = \sqrt{25 \times 5} = 5\sqrt{5}$

$= 5 \times (2.236) = 11.18$ cm

$\therefore$ Distance between their centres = 11.18 cm.

10. Let PQ = 7 cm, QR = 8 cm and RP = 11 cm.

Let x, y, z be the radii of the three circle.



Then, $x + y = 7$ ...(*i*)

$y + z = 8$ ...(*ii*)

$z + x = 11$ ...(*iii*)

Adding (*i*) (*ii*) and (*iii*), we have

$2(x + y + z) = 26 \Rightarrow x + y + z = \frac{26}{2} = 13$ ...(*iv*)

Subtracting (*i*) from (*iv*), we have

$(x + y + z) - (x + y) = 13 - 7 = 6 \Rightarrow z = 6$

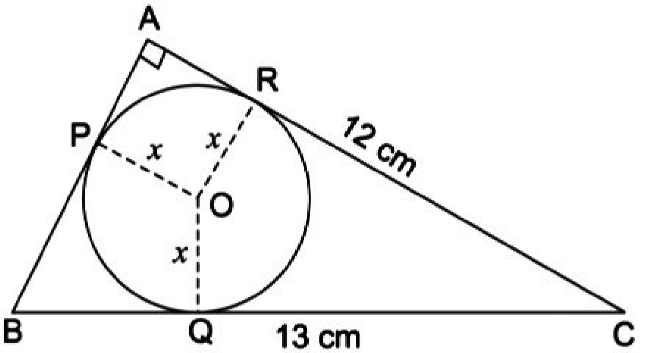
Similarly

$\{(x + y + z) - (y + z)\} = 13 - 8 = 5 \Rightarrow x = 5$

$\{(x + y + z) - (z + x)\} = 13 - 11 = 2 \Rightarrow y = 2$

Hence, radii of the three circle will be 5 cm, 2 cm and 6 cm.

11. ΔBAC is a right– angled triangle, right angle at A, $AC = 12$ cm BC = 13 cm. *A* circle with centre *O* is drawn in the triangle touching its sides at *P, Q,* R respectively.



Now, in right ΔBAC, we have

$BC^2 = AC^2 + AB^2$ [Pythagoras Theorem]

$\Rightarrow (13)^2 = (12)^2 + AB^2$

$\Rightarrow 169 = 144 + AB^2 \Rightarrow AB^2 = 25 = (5)^2$

$\Rightarrow AB = 5$ cm

Now it is clear that $APOR$ is square, where each side is x.

$\because$ CQ and CR are the tangents

$\therefore$ CQ = CR

BP and BQ are the tangents.

$\therefore$ BP = BQ

Now, CR = CA – AR

$\Rightarrow$ CR = 12 – x $\Rightarrow$ CQ = 12 – x ...(i)

Also, BP = AB – AP

$\Rightarrow$ BP = 5 – x

$\Rightarrow$ BQ = 5 – x ...(ii)

Adding (i) and (ii), we have

$$CQ + BQ = 12 - x + 5 - x$$

$$BC = 17 - 2x$$

$$\Rightarrow \quad 13 = 17 - 2x \Rightarrow 2x = 17 - 13 = 4$$

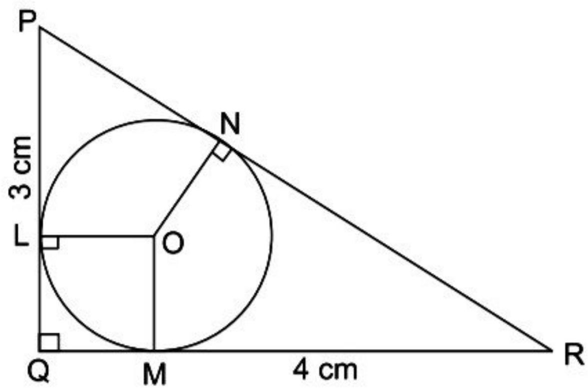
$$\therefore \quad x = \frac{4}{2} = 2$$

Hence, value of x = 2 cm.

12. In ΔPQR, $\angle$Q = 90° and PQ = 3 cm, QR = 4 cm

$$\therefore PR = \sqrt{PQ^2 + QR^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5\,\text{cm}$$

A circle with centre O, is drawn which touches the ΔPQR at L, M, and N respectively.



Let O be the centre of the circle OL, OM, ON are joined.

Clearly, QLOM is a square and let OL = OM = r

$\therefore$ RM = (4 – r) cm and PL= (3 – r) cm

But RM = RN and PL = PN [$\because$ Tangents from the outer points to the circle]

$\therefore$ RN = 4 – r and PN = 3 – r.

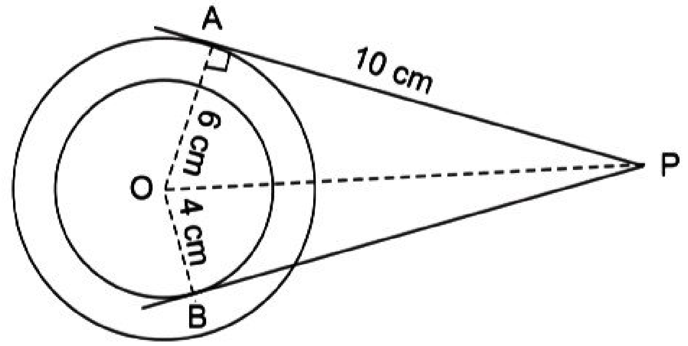
But PR $= 5$ cm

$\therefore\ 5 = 7 - 2r$

$\Rightarrow 2r = 7 - 5 = 2 \quad \Rightarrow \quad r = \frac{2}{2} = 1$

Hence radius of the incircle is 1 cm.

13. Two concentric circles with centre *O* and radius OA and OB respectively. P and BP are the tangents drawn from P to the circles. Join OA, OB and OP.0 AP = 10 cm, OA = 6 cm, OB = 4 cm



$\because AP$ is tangent and OA is radius

$\therefore OA \perp AP$

Similarly, $OB \perp BP$

Now, in right ΔOAP, we have

$$OP^2 = OA^2 + AP^2 = (6)^2 + (10)^2 \quad \text{[Pythagoras Theorem]}$$

$$= 36 + 100 = 136 \quad ...(i)$$

Similarly, in right ΔOBP, we have

$$OP^2 = OB^2 + PB^2 = (4)^2 + PB^2 \quad \text{[Pythagoras Theorem]}$$

$$= 16 + PB^2 \quad ...(ii)$$

From (*i*) and (*ii*), we have $136 = 16 + PB^2$

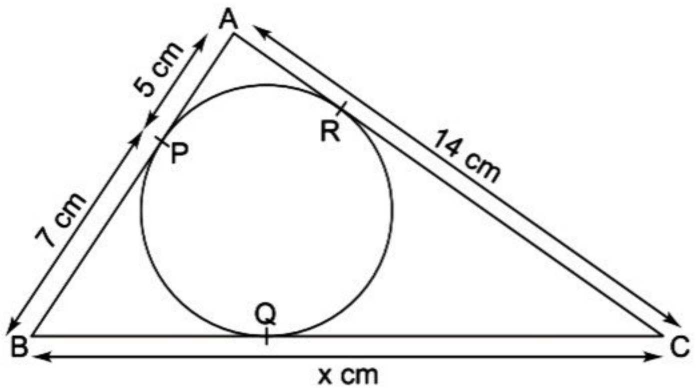
$\Rightarrow PB^2 = 136 - 16 = 120$

$\Rightarrow PB = \sqrt{120}$ cm $= 10.95$ cm.

14. ΔABC is circumscribed and circle touches its sides *AB, BC, CA*, at *P, Q* and *R* respectively.

$$AP = 5 \text{ cm}, BP = 7 \text{ cm}, AC = 14 \text{ cm and BC} = x$$

From *A, AP* and *AR* are the tangents to the circle.



∴ $AP = AR \Rightarrow AR = 5$ cm.

∴ $CR = 14$ cm – 5 cm = 9 cm

Now from C, CR and CQ are the tangents.

∴ $CR = CQ \Rightarrow CQ = 9$ cm

Now, from *B, BQ* and *BP* are the tangents.

∴ $BP = BQ \Rightarrow BQ = 7$ cm.

∴ $BC = BQ + CQ = 7 + 9 = 16$ cm.

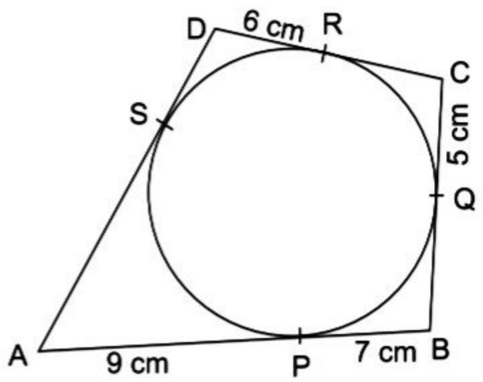
Hence, $x = 16$ cm.

15. Quadrilateral ABCD is circumscribed. A circle touches its sides AB, BC, CD and DA at P, Q, R and S respectively.

AP = 9 cm, BP = 7 cm, CQ = 5 cm and DR = 6 cm

∵ From A, AP and AS are the tangents to the circle.

∴ AP = AS = 9 cm.



Similarly, BP = BQ = 7 cm.

CQ = CR = 5 cm.

And DR = DS = 6 cm.

Now, AB = 9 + 7 = 16 cm.

BC = 7 + 5 = 12 cm.

CD = 5 + 6 = 11 cm

and DA = 6 + 9 = 15 cm.

∴ Perimeter of quadrilateral ABCD

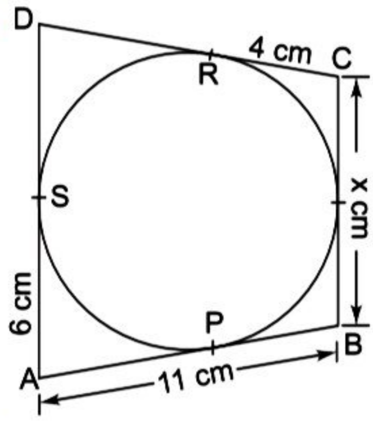
= AB + BC + CD + DA

= (16 + 12 + 11 + 15)cm = 54 cm.

16. The given circle touches the sides *AB, BC CA* and *DA* at *P, Q* , *R* and *S* respectively.

$AB = 11$ cm , $BC = x$ cm, $CR = 4$ cm and $AS = 6$ cm.

$\because$ From A, AP and AS are the tangents to the circle, therefore $AP = AS = 6$ cm.



$\therefore\ BP = AB - AP = (11 - 6)$ cm

$= 5$ cm.

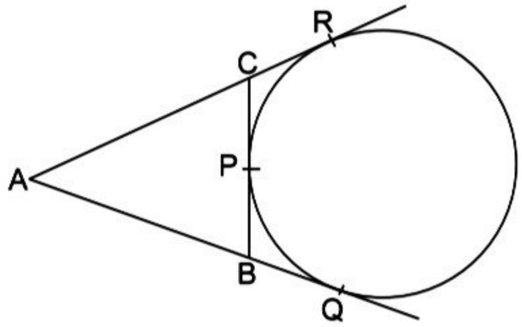
Similarly, $BP = PQ = 5$ cm

and $CQ = CR = 4$ cm

Now, $BC = BQ + CQ = BP + CR = 5 + 4 = 9$ cm.

Hence, $x = 9$ cm.

17. From the figure, a circle touches the sides AB and AC produced at Q and R internally and BC at P externally. $AQ = 15$ cm.



$\because$ From A, AQ and AR are the tangents to the circle

$\therefore AR = AQ = 15$ cm

Now, perimeter of $\Delta ABC = AB + AC + BC$

$= AB + AC + BP + CP = AB + AC + BQ + CR$

$= (AB + BQ) + (AC + CR) = AQ + AR$

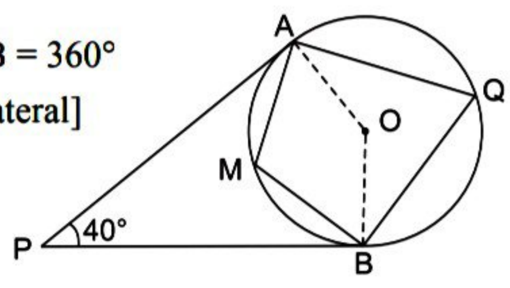
$= 15 + 15 = 30$ cm.

18. From the figure, PA and PB are two tangents to the circle with centre O. $\angle APB = 40°$

Join OA and OB.

Now, $\angle OAP = 90°$ [$\because$ OA is radius and PA is tangent]

Similarly, $\angle OBP = 90°$



But, $\angle OAP + \angle APB + \angle PBO + \angle AOB = 360°$

[Sum of angles of a quadrilateral]

$\Rightarrow 90° + 40° + 90° + \angle AOB = 360°$

$\Rightarrow 220° + \angle AOB = 360°$

$\Rightarrow \angle AOB = 360° - 220°$

$\therefore \angle AOB = 140°$

$\therefore \angle AOB = 2\angle AQB$

$\Rightarrow \angle AQB = \frac{1}{2}\angle AOB \Rightarrow \angle AQB = \frac{1}{2} \times 140° = 70°$

$\because$ *AMB*Q is a cyclic quadrilateral.

$\therefore \angle AMB + \angle AQB = 180°$

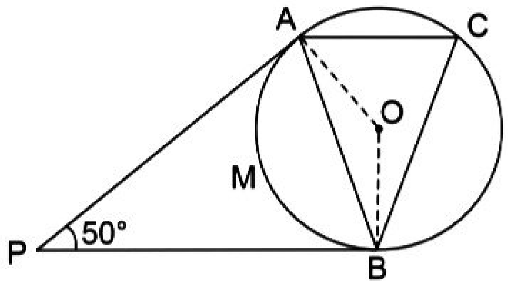
$\Rightarrow \angle AMB + 70° = 180°$

$\Rightarrow \angle AMB = 180° - 70° = 110°$

19. PA and PB are the tangents to the circle with center O. $\angle APB = 50°$

(*i*) $\because$ *OA* is a radius and AP is the tangent to the circle.

$\therefore OA \perp AP$



Similarly, $OB \perp BP$

Now, $\angle OAP + \angle APB + \angle OBP + \angle AOB = 360°$

[Sum of angles of a quadrilateral]

$\Rightarrow 90° + 50° + 90° + \angle AOB = 360°$

$\Rightarrow 230° + \angle AOB = 360°$

$\Rightarrow \angle AOB = 360° - 230°$

$\therefore \angle AOB = 130°$

(*ii*) In ΔOAB, $OA = OB$ [Radii of the same circle]

$\therefore \angle OAB = \angle OBA$

Now, $\angle OAB + \angle OBA + \angle AOB = 180°$ [Sum of angles in a triangle]

$\Rightarrow \angle OAB + \angle OAB + 130° = 180°$ [$\therefore \angle OAB = \angle OBA$]

$\Rightarrow 2\angle OAB = 180° - 130° = 50°$

$\therefore \angle OAB = \frac{50\Upsilon}{2} = 25°$

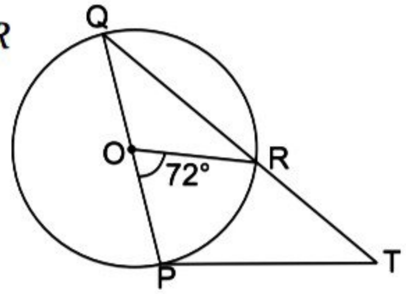
(*iii*) Now, arc *AB* subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle.

$\therefore \angle AOB = 2\angle ACB$

$\Rightarrow \angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 130° = 65°$

meets the circle at R. $\angle POR = 72°$

Arc PR subtends $\angle POR$ at the centre and $\angle PQR$ at the remaining part of the circle.



$\therefore \quad \angle \text{POR} = 2\angle \text{PQR}$

$\Rightarrow \quad \angle \text{PQR} = \frac{1}{2} \angle \text{POR}$

$\therefore \quad \angle \text{PQR} = \frac{1}{2} \times 72° = 36°$

Now in ΔQPT, we have

$\angle QPT + \angle PTQ + \angle PQT = 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad 90° + 36° + \angle PTQ = 180°$

$\Rightarrow \quad 126° + \angle \text{PTQ} = 180°$

$\Rightarrow \quad \angle PTQ = 180° - 126°$

$\therefore \quad \angle PTQ = 54°$ or $\angle PTR = 54°$

21. O is the centre of the circumcircle of ΔABC . At A and B, tangents AT and BT are drawn to meet at T.

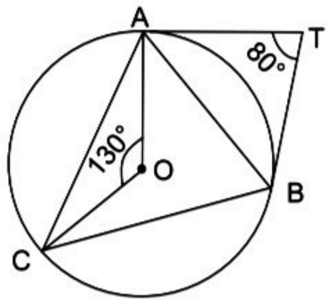
$\angle ATB = 80°$ and $\angle AOC = 130°$

$\because \quad TA = TB$ [Tangents from T]

$\therefore \quad \angle TAB = \angle TBA$

Now in ΔTAB, we have

$\angle TAB + \angle TBA + \angle ATB = 180°$ [Sum of angles of a triangle]



$\Rightarrow \angle TAB + \angle TAB + 80° = 180°$ [$\because \angle$TAB = $\angle$ TBA]

$\Rightarrow 2\angle TAB = 180° - 80° = 100°$

$\therefore \angle TAB = \frac{100\Upsilon}{2} = 50°$

$\because OA = OC$ [Radii of the same circle]

$\angle OAC = \angle OCA$

Now in ΔOAC, we have

$\angle OAC + \angle OCA + \angle AOC = 180°$ [Sum of angles of a triangle]

$\Rightarrow \angle OAC + \angle OAC + 130° = 180°$ [$\therefore \angle$OAC = $\angle$ OCA]

$\Rightarrow 2\angle OAC = 180° - 130° = 50° \Rightarrow \angle OAC = \frac{50\Upsilon}{2} = 25°$ [$\because \angle$OAC = $\angle$OCA]

$\because$ OA is radius and AT is the tangent.

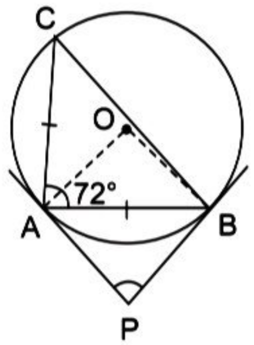
$\therefore \angle OAT = 90°$

Now, $\angle CAB = \angle CAO + \angle OAT - \angle TAB$

$= 25° + 90° - 50° = 65°$

22. From the given figure, PA and PB are tangents to the circle with centre O. ΔABC is inscribed in circle such that $AB = AC$, $\angle BAC = 72°$

Now in ΔABC, we have



$\Rightarrow \quad \angle ABC + \angle ACB = 180° - 72° = 108°$

But $\quad \angle ABC = \angle ACB$ [Angle opposite to equal sides]

$\therefore \quad \angle ACB + \angle ACB = 108°$

$\Rightarrow \quad 2\,\angle ACB = 108° \quad \Rightarrow \quad \angle ACB = \dfrac{108\Upsilon}{2} = 54°.$

(*i*) Arc AB, subtends $\angle AOB$ at the centre and $\angle ACB$ on the remaining part of the circle.

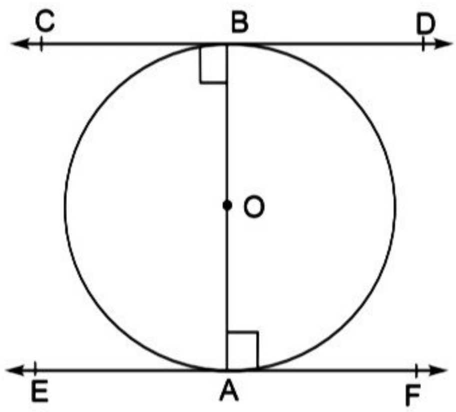
$\therefore \angle AOB = 2\,\angle ACB = 2 \times 54° = 108°$

(*ii*) $\angle APB = 180° - \angle AOB$

$= 180° - 108° = 72°$

23. Given: AB is the diameter of the circle with centre O. At A and B, tangents EAF and CBD are drawn.

To prove . $CD \parallel EF$



Proof. $\because$ OA is radius and EAF is the tangent.

$\therefore OA \perp EF$ or $\angle OAE = 90°$ ...(*i*)

Again, OB is radius and CBD is the tangent. Therefore,

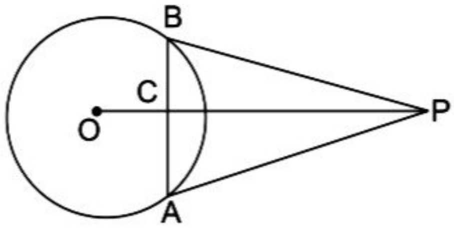
$\angle OBD = 90°$ .....(*ii*)

From (i) and (ii), $\angle OAE = \angle OBD$

But these are alternate angles,

$\therefore CD \parallel EF$. **Hence Proved**

24. AB is the chord of the circle with centre O. BP and AP are the tangents drawn meeting each other at P. OP is joined intersecting AB at C.



To Prove. $\angle$PAC = $\angle$PBC

Proof: In ΔPAC and ΔPBC,

$PA = PB$ [Tangents from P]

$PC = PC$ [Common]

$\angle APC = \angle BPC$ [OP bisects $\angle$APB]

$\therefore \Delta PAC \cong \Delta PBC$ [S.A.S. axiom]

Hence, $\angle PAC = \angle PBC$ [C.P. C. T.]

25. Given: AB and CD are two tangents such that AB $\parallel$ CD. PO and QO are joined.

To Prove. POQ is a straight line.

Construction: Draw $OE \parallel AB \parallel CD$.

Proof. $\because$ OP is the radius and AB is the tangent.

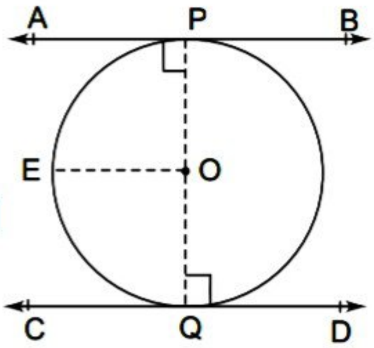
$\therefore \angle OPA = 90°$

Similarly, $\angle OQC = 90°$

$\because$ OE $\parallel$ AB

$\therefore \angle OPA + \angle POE = 180°$

[Angles on the same side of the transversal]



$\Rightarrow 90° + \angle POE = 180°$

$\Rightarrow \angle POE = 180° - 90° = 90°$

Similarly,OE $\parallel$ CD

$\because \angle QOE + \angle OQC = 180°$

$\Rightarrow \angle QOE + 90° = 180°$

$\Rightarrow \angle QOE = 180° - 90° = 90°$

$\because \angle POE + \angle QOE = 90° + 90° = 180°$

Hence, POQ is a straight line. **Hence proved.**

26. PQ is a transverse common tangent to the two circles with centre A and B

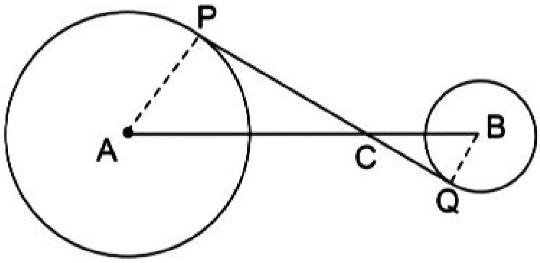
respectively. The radii of circles are 5 cm and 3 cm . AB is joined which intersects PQ at C and $CP = 12$ cm. Join AP and BQ.

$\because$ AP is radius and PQ is tangent.

$\therefore \angle APQ = 90°$

Similarly, $\angle BQC = 90°$

Now, in ΔPAC and ΔQBC, we have



$\angle APC = \angle BQC$ [Each 90°]

$\angle PCA = \angle QCB$ [Vertically opposite angles]

$\therefore \Delta PAC \sim \Delta QBC$ [AA axiom]

$\therefore \frac{AC}{CB} = \frac{PC}{CQ} = \frac{AP}{BQ}$

$\Rightarrow \frac{PC}{CQ} = \frac{AP}{BQ} \Rightarrow \frac{12}{CQ} = \frac{5}{3}$

$\Rightarrow CQ = \frac{12 \times 3}{5} = \frac{36}{5}$ cm $= 7.2$ cm.

Now, in ΔAPC we have

$AC^2 = PC^2 + AP^2$ [Pythagoras Theorem]

$AC^2 = 12^2 + 5^2 = 144 + 25 = 169 = (13)^2$

$\therefore AC = 13$ cm.

Similarly, in right ΔBCQ , we have

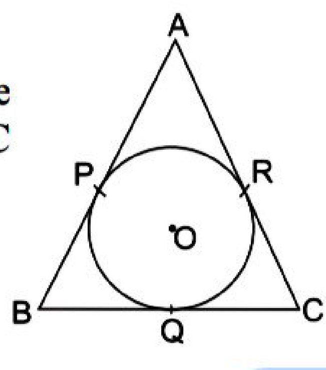
$BC^2 = QC^2 + QB^2$ [Pythagoras Theorem]

$= (7.2)^2 + (3)^2 = 51.84 + 9 = 60.84 = (7.8)^2$

$\therefore BC = 7.8$ cm.

Hence, $AB = AC + CB = (13 + 7.8)$ cm $= 20.8$ cm.

27. **Given:** ΔABC circumscribed about a circle with centre O. $AB = AC$ and the circle touches the sides AB, BC and CA at P, Q and R respectively.



To Prove. Q bisects BC.

Proof. AP and AR are the tangents to the circle.

Similarly, $BP = BQ$ and $CQ = CR$

$\because \quad AB = AC$ and $AP = AR$

$\therefore \quad AB - AP = AC - AR$

$\Rightarrow \quad BP = CR$

But $\quad BQ = BP$ and $CQ = CR$

$\therefore \quad BQ = CQ$

Hence Q is the mid - point of BC. **Hence proved.**

28. From the figure, quadrilateral ABCD is circumscribed about a circle with centre O. AD $\perp$ AB. Radius of circle = 10 cm. AB = x cm.

BC = 38 cm , CR = 27 cm.

$\because$ DR and DS are the tangents to the circle from D.

$\therefore$ $DR = DS = y$

$\because$ $OS \perp AD$ and $OP \perp AB$

$\therefore$ APOS is a square

$\therefore$ AS = OS = 10 cm

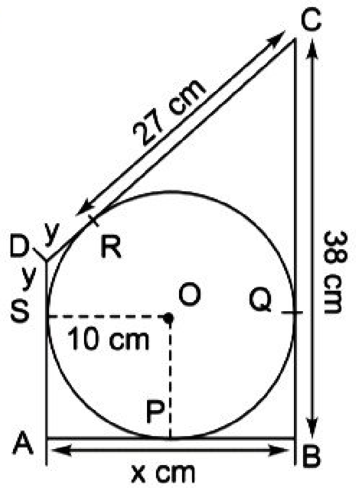
$\because$ The circle touches the sides of the quadrilateral.

$\therefore$ AB + CD = AD + BC

$\Rightarrow x + 27 + y = y + 10 + 38.$

$\Rightarrow x = y + 10 + 38 - 27 - y = 21$

Hence, $x = 21$ cm.



29. From the figure a circle with centre O is inscribed in a quadrilateral ABCD DC = 25 cm, CB = 38 cm. BQ = 27 cm. AD $\perp$ DC.

$\because$ BQ and BR are the tangents to the circle from B

$\therefore$ $BR = BQ = 27$ cm

$\therefore$ $CR = BC - BR = 38 - 27 = 11$ cm.

Similarly, $CS = CR = 11$ cm.

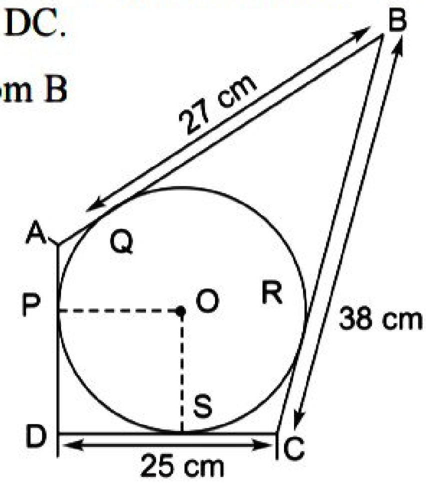
$\therefore$ $DS = DC - CS = 25 - 11 = 14$ cm.

$\because$ $OP \perp AD$ and $OS \perp DC$

$\therefore$ DSOP is a square

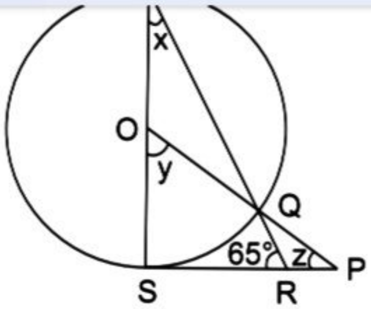
$\therefore$ DS = PO = radius of the circle

$\therefore$ Radius of the circle = 14 cm.



30. **Given:** O is the centre of the circle.

SP is the tangent to the circle at S.



To find: The value of x, y and z

Proof $\because SP$ is tangent to the circle and OS is the radius.

$\therefore \angle OSP = 90° \quad \Rightarrow \quad \angle TSP = 90°$

In ΔTSR,

$\angle STR + \angle TSR + \angle TRS = 180°$ [Sum of angles of a triangle]

$\Rightarrow x° + 90° + 65° = 180°$

$\Rightarrow x° + 155° = 180°$

$\Rightarrow x° = 180° - 155° = 25°$

Arc SQ subtends $\angle SOQ$ at the centre of the circle and $\angle STQ$ at the remaining part of the circle.

$\therefore \angle SOQ \quad = 2 \angle STQ$

$\Rightarrow y \quad = 2x \quad \Rightarrow \quad y = 2 \times 25° = 50°$

In ΔOSP, we have:

$\angle OSP + \angle SOP + \angle SPO = 180°$ [Sum of angles of a triangle]

$90° + 50° + z° = \quad 180°$

$\therefore \quad z° = 180° - (90° + 50°)$

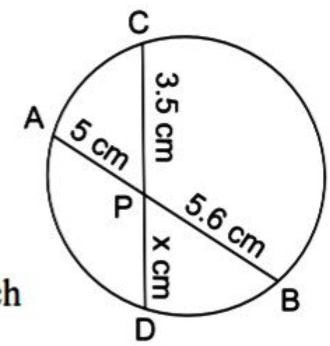
$\Rightarrow \quad z° = 180° - 140° = 40°$

$\therefore \quad x° = 25°, y° = 50°$ and $z° = 40°$

EXERCISE 19B

1. (i) From the figure, chords AB and CD intersect each other at P inside the circle.

$\therefore AP \times PB = CP \times PD$



$\Rightarrow 5 \times 5.6. = 3.5 \times x$

$\Rightarrow x = \dfrac{5 \times 5.6}{3.5} = 8$

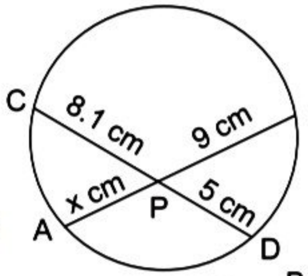
$\therefore x = 8$ cm.

(ii) From the figure chords AB and CD intersect each other at P inside the circle.

$\therefore \quad AP \times PB = CP \times PD$

$\Rightarrow x \times 9 = 8.1 \times 5 \quad \Rightarrow \quad x = \dfrac{8.1 \times 5}{9}$

$\therefore \ x = 4.5$ cm



(*iii*) From the figure chords *AB* and *CD* intersect each other at *P* outside the circle.

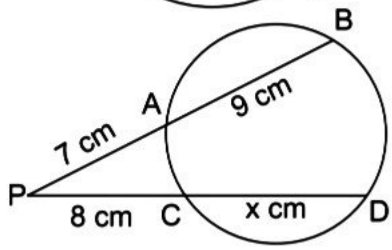
$\therefore AP \times PB = CP \times PD$

$7 \times (7 + 9) = 8\,(8 + x)$

$\Rightarrow 7 \times 16 = 8\,(8 + x)$

$\Rightarrow 8\,(8 + x) = 112 \quad \Rightarrow \quad 8 + x = \dfrac{112}{8} = 14$

$\therefore x = 14 - 8 = 6$ cm



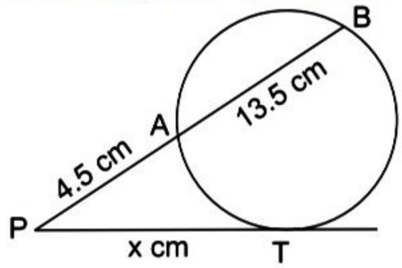
(*iv*) From the figure, *PAB* is the secant and *PT* is the tangent to the circle

$\therefore \quad PT^2 = PA \times PB$

$\Rightarrow \quad x^2 = 4.5\,(4.5 + 13.5)$

$= 4.5 \times 18 = 81$

$\Rightarrow x = \sqrt{81} = 9$ cm



(*v*) From the figure, *PAB* is the secant and *PT* is the tangent to the circle.

$\therefore PT^2 = PA \times PB$

$\Rightarrow \quad (12)^2 = x \times (x + 10)$

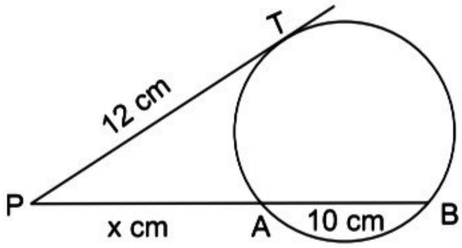
$\Rightarrow \quad 144 = x^2 + 10x$

$\Rightarrow \quad x^2 + 10x - 144 = 0$

$\Rightarrow \quad x^2 + 18x - 8x - 144 = 0$

$\Rightarrow \quad x(x + 18) - 8\,(x + 18) = 0$

$\Rightarrow \quad (x + 18)\,(x - 8) = 0$ [Zero product rules]



Either $x + 18 = 0$, then $x = -18$ which is not possible.

or $x - 8 = 0$, then $x = 8$

Hence, $x = 8$ cm

2. Since, PT is a tangent and ABP is the secant.

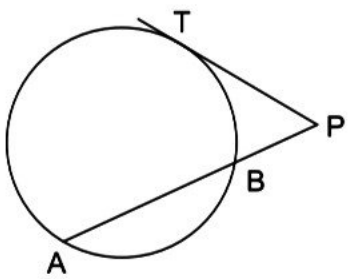
We know that,

PB = PA – AB = 16 – 12 = 4 cm

[$\therefore$ PA = 16 cm, A B = 12 cm]

Now, $PT^2 = PA \times PB = 16 \times 4 = 64\ cm^2$

$\therefore PT = \sqrt{64\ cm^2} = 8$ cm

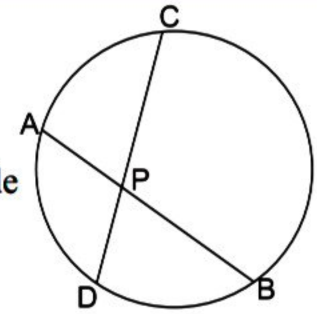


3. $AB = 12$ cm, $AP = 2.4$ cm

$\therefore PB = AB - AP = 12 - 2.4 = 9.6$ cm

Let $CP = x$

$\because$ Chords AB and CD intersect each other at P inside the circle.



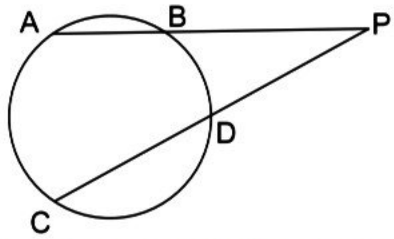
$\therefore \quad AP \times PB = CP \times PD$

$\Rightarrow \quad 2.4 \times 9.6 = x \times 7.2$

$\Rightarrow \quad x = \dfrac{2.4 \times 9.6}{7.2} = 3.2\text{ cm} \quad \Rightarrow \quad \text{CP} = 3.2\text{ cm}$

Hence, CD = CP + PD = 3.2 + 7.2 = 10.4 cm.

4. PA = 12 cm, AB = 4 cm



$\therefore$ BP = AP – AB

= 12 cm – 4 cm = 8 cm.

CD = 10 cm.

Let PD = x

$\therefore \quad CP = (10 + x)$ cm.

$\because$ Two Chords AB and CD intersect each other at P outside the circle.

$\therefore \quad PA \times PB = PC \times PD$

$\Rightarrow 12 \times 8 = (10 + x) \times x \quad \Rightarrow \quad 96 = 10x + x^2$

$\Rightarrow x^2 + 10x - 96 = 0 \quad \Rightarrow \quad x^2 + 16x - 6x - 96 = 0$

$\Rightarrow x(x + 16) - 6(x + 16) = 0$

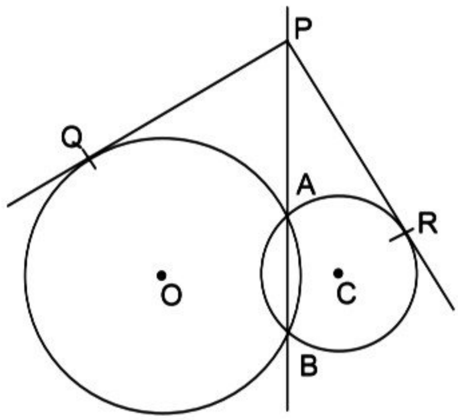
$\Rightarrow (x + 16)(x - 6) = 0$ [Zero product rule]

Either $x + 16 = 0$, then $x = -16$ which is not possible

or $x - 6 = 0$, then $x = 6$

Hence, PD = 6 cm .

5.



Given: Two circles with centre O and C intersect each other at A and B . P is a point on BA produced and from P, PQ and PR are tangents to these circles.

To Prove: $PQ = PR$

Proof: $\because$ PQ is the tangent and PAB is the secant of the circle with centre O.

$\therefore \quad PA \times PB = PQ^2$ ...(*i*)

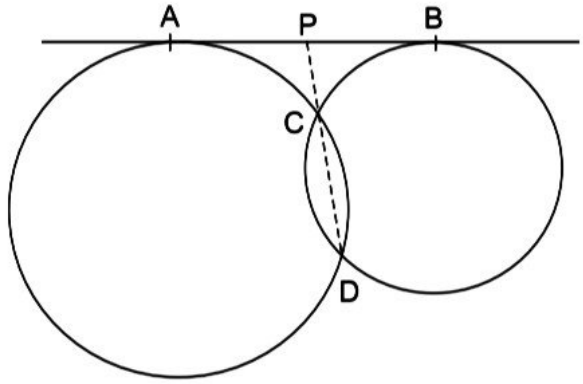
Similarly, PR is the tangent and PAB is the secant of the circle with centre C.

$\therefore \quad PA \times PB = PR^2$ ...(*ii*)

From (*i*) and (*ii*), we have

$$PQ^2 = PR^2 \Rightarrow PQ = PR.$$ **Hence proved.**

6.



Given: AB is the direct common tangent to the circles which intersect each other at C and D. DC is produced to meet AB at P.

To prove: P is mid-point of AB.

Proof: $\because$ PA is tangent and PCD is the secant to the first circle

$\therefore PA^2 = PC \times PD$ ....(*i*)

Again PB is the tangent and PCD is the secant of the second circle.

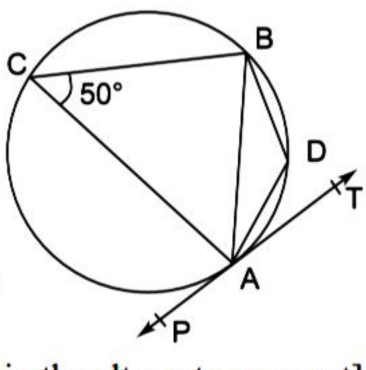
$\therefore PB^2 = PC \times PD$ ....(*ii*)

From (*i*) and (*ii*), we have

$$PA^2 = PB^2 \Rightarrow PA = PB$$

Hence, P is the mid- point of AB.

7. From the figure, PAT is tangent to the circle at A. ΔABC is inscribed in the circle and $\angle ACB = 50°$



(*i*) $\because$ PAT is the tangent and AB is the chord of the circle.

$\therefore \angle ACB = \angle BAT$ [Angles in the alternate segment]

$\therefore \angle TAB = 50°$

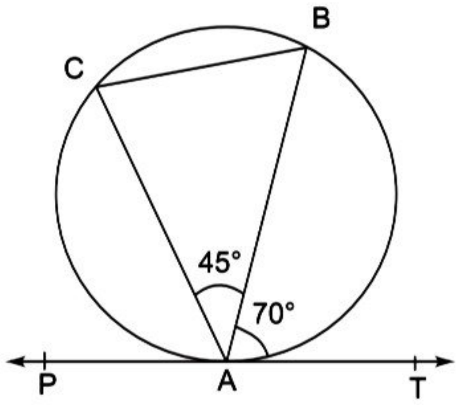
(*ii*) ADBC is a cyclic quadrilateral

$\therefore \quad \angle ADB + \angle ACB = 180°$

$\Rightarrow \quad \angle ADB + 50° = 180°$

$\Rightarrow \quad \angle ADB = 180° - 50° = 130°.$

8. $\because$ *PTA* is the tangent and *BA* is the chord of the circle.



$\therefore \angle ACB = \angle BAT = 70°$ [Angles in the alternate segment]

Now in ΔABC, we have

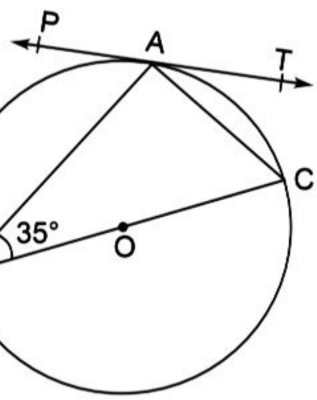
$\angle ABC + \angle BCA + \angle BAC = 180°$ [Sum of angles in a triangle]

$\Rightarrow \quad \angle ABC + 70° + 45° = 180°$

$\Rightarrow \quad \angle ABC + 115° = 180°$

$\Rightarrow \quad \angle ABC = 180° - 115° = 65°$

9. (*i*) $\because$ *PAT* is the tangent and *AC* is the chord of the circle.



$\therefore \quad \angle TAC = \angle ABC = 35°$ [Angles in the alternate segment]

[$\because \angle ABC = 35°$]

(*ii*) $\angle BAC = 90°$ [Angle in semi-circle]

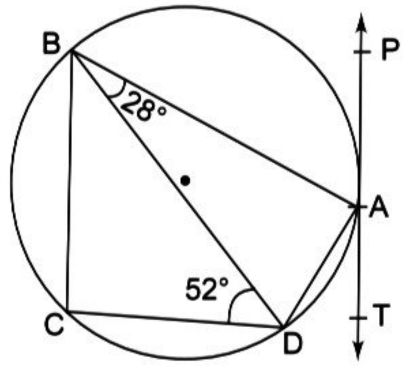
$\angle PAB + \angle BAC + \angle TAC = 180°$

$\Rightarrow \angle PAB + 90° + 35° = 180°$

$\Rightarrow \angle PAB + 125° = 180°$

$\Rightarrow \angle PAB = 180° - 125° = 55°$

10. *(i)* *PAT* is the tangent and *AD* is the chord of the circle.



$\therefore \quad \angle TAD = \angle ABD = 28°$ [Angles in the alternate segment]

(ii) $\because$ BD is the diameter of the circle

$\therefore \quad \angle BAD = 90°$ [Angles in a semi-circle]

(*iii*) $\angle PAB = \angle ADB$ [Angles in the alternate segment]

But, $\angle ADB = 180° - (\angle ABD + \angle BAD)$ [Angles of a triangle]

$= 180° - (28° + 90°)$

$= 180° - 118° = 62°$

$\therefore \quad \angle ADB = 62° \Rightarrow \angle PAB = 62°$

(*iv*) In ΔBCD, we have

$\angle CBD + \angle BCD + \angle BDC = 180°$ [Angle sum of a triangle]

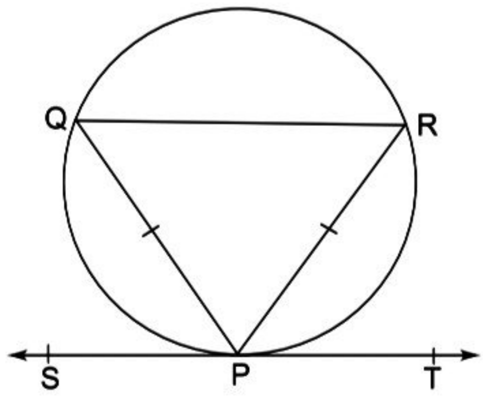
$\Rightarrow \angle CBD + 90° + 52° = 180°$[$\because$ $BCD = 90°$ Angle in a semi-circle]

$\Rightarrow \angle CBD + 142° = 180°$

$\Rightarrow \angle CBD = 180° - 142° = 38°$

Hence, $\angle CBD = 38°$

11. **Given:** PQ and PR are two equal chords of the circle. *QR* is joined and SPT is the tangent.



To Prove. $QR \parallel SPT$

Proof $\because PQ = PR$ [Given]

$\therefore \quad \angle PRQ = \angle PQR$ [Equal arcs subtend equal angles at the circumference]

But, $\angle RPT = \angle PQR$ [Angles in the alternate segment]

$\therefore \quad \angle PRQ = \angle RPT$

But these are alternate angles.

$\therefore QR \parallel SPT$. **Hence proved**

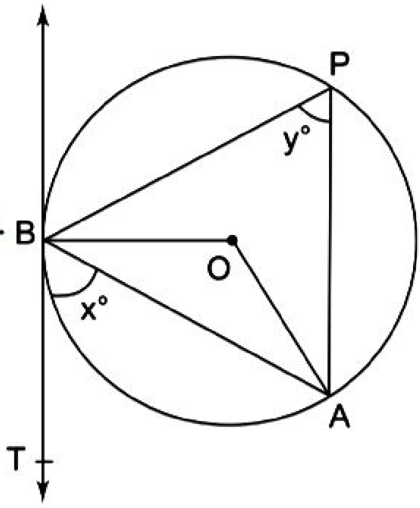
12. *AB* is the chord of the circle with centre *O* and *BT* is the tangent.

$\angle OAB = 35°$.

$\therefore \angle ABT = \angle APB$

[Angles in the alternate segment]

$\Rightarrow x° = y°$



In ΔOAB, $OA = OB$ [Radii of the same circle]

$\therefore \angle OAB = \angle OBA = 35°$

But $\angle OAB + \angle OBA + \angle AOB = 180°$

$\Rightarrow 35° + 35° + \angle AOB = 180°$

$\Rightarrow 70° + \angle AOB = 180°$

$\Rightarrow \angle AOB = 180° - 70° = 110°$

Now, arc AB subtends $\angle AOB$ at the centre and $\angle APB$ at the remaining part of the circle.

$\therefore \angle AOB = 2\angle APB$

$\Rightarrow 110° = 2y° \Rightarrow y° = \frac{110Υ}{2} = 55°$

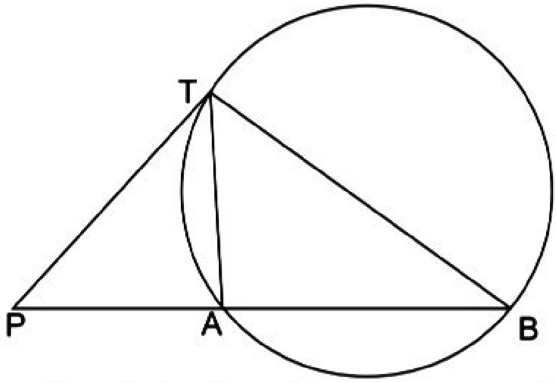
Hence, $x° = y° = 55°$

13. **Given:** *PAB* is the secant to a circle and *PT* is the tangent. *AT* is joined.

To Prove:

(*i*) $\Delta PAT \sim \Delta PTB$

(*ii*) $PA \times PB = PT^2$



Proof. (i) ΔPAT and ΔPTB

$\angle P = \angle P$ [Common]

$\angle PTA = \angle ABT$ or $\angle PBT$

[Angle in the alternate segment]

$\therefore \Delta PAT \sim \Delta PTB$ [*AA* similarity axiom]

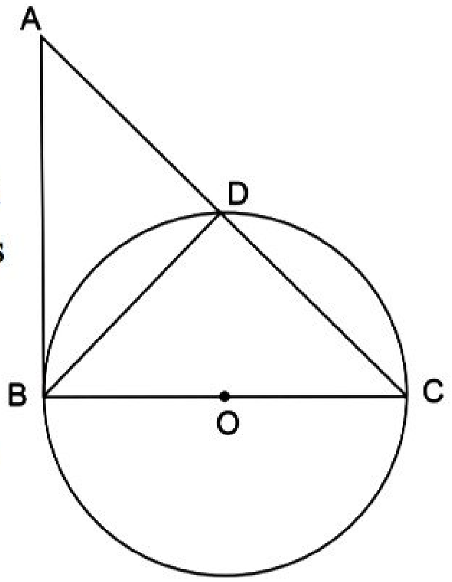
(*ii*) $\because \Delta PAT \sim \Delta PTB$ [Proved in (*i*)]

$\therefore \quad \frac{\quad}{PB} = \frac{\quad}{PT}$

$\Rightarrow \quad PT \times PT = PA \times PB$

$\Rightarrow \quad PT^2 = PA \times PB$

14. Given ΔABC is a right- angled at D. BD is a perpendicular on AC.



To Prove:

(*i*) $AC \times AD = AB^2$

(*ii*) $AC \times CD = BC^2$

Construction: Draw a circumcircle of ΔBCD.

Proof. (*i*) $\because$ AB is the tangent and ADC is a secant of the circle.

$\therefore \quad AB^2 = AC \times AD$

(*ii*) $AC \times CD = AC \times (AC - AD)$

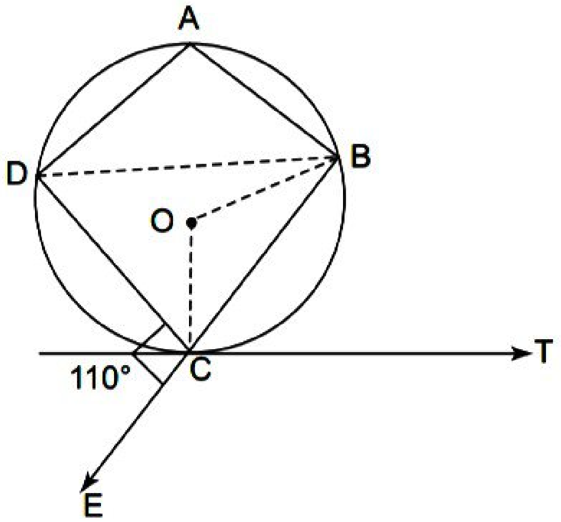
$= AC^2 - AC \times AD = AC^2 - AB^2$

But in right ΔABC,

$AC^2 = AB^2 + BC^2 \quad \Rightarrow \quad AC^2 - AB^2 = BC^2$

$\therefore \quad AC \times CD = BC^2$ **Hence proved.**

15.



(*i*) *ABCD* is a cyclic quadrilateral $CB = CD$ and *TC* is the tangent to the circle at *C*. BC is produced to E, $\angle DCE = 110°$.

Join *BD*, *OB* and *OC*.

$\because$ *BCE* is a straight line.

$\therefore \quad \angle BCD + \angle DCE = 180°$ [Linear Pair]

$\Rightarrow \quad \angle BCD + 110° = 180°$

$\Rightarrow \angle BCD = 180° - 110°$

$\Rightarrow \angle BCD = 70°$

Now in the ΔBCD,

$\because$ $BC = CD$ [Given]

$\therefore$ $\angle BDC = \angle DBC$ [Angles opposite to equal sides]

$\therefore$ $\angle BDC = \angle DBC = \dfrac{110°}{2} = 55°$

$\angle BCT = \angle DBC$ [Angle in the alternate segment]

$\Rightarrow$ $\angle BCT = 55°$ [$\because \angle BDC = 55°$]

$\therefore$ $\angle DCT = \angle DCB + \angle BCT$

$= \angle BCD + 55°$

$= 70° + 55°$

$= 125°$

Hence, $\angle DCT = 125°$

(*ii*) Arc BC subtends $\angle BOC$ at the centre and $\angle BDC$ at the remaining part of the circle.

$\therefore$ $\angle BOC = 2\ \angle BDC = 2 \times 55° = 110°$