18

ANGLE AND CYCLIC PROPERTIES OF A CIRCLE

EXERCISE 18

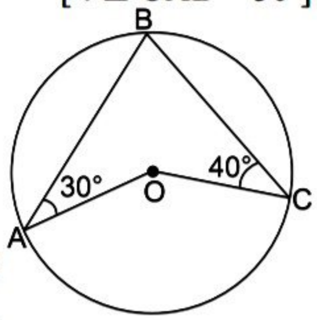
1. From the figure join OB.

In ΔAOB, we have

$OA = OB$ [Radii of the same circle]

$\therefore \quad \angle OAB = \angle OBA$ [Opposite angles to equal sides]

$\therefore \Rightarrow \angle OBA = 30°$ [$\because \angle OAB = 30°$]



Similarly, in ΔOBC, we have $OB = OC$

$\therefore \Rightarrow \angle OBC = \angle OCB = 40°$

Adding we get :

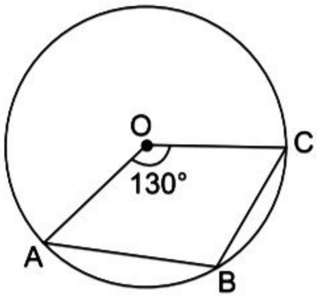
$\angle OBA + \angle OBC = 30° + 40° = 70°$

Now, arc AC subtends $\angle AOC$ at the centre of the circle and $\angle ABC$ at the remaining part of the circle.

$\therefore \quad \angle AOC = 2\angle ABC = 2 \times 70° = 140°.$

2. From the figure, $\angle AOC = 130°$

$\therefore$ Reflex $\angle AOC = 360° - 130° = 230°$



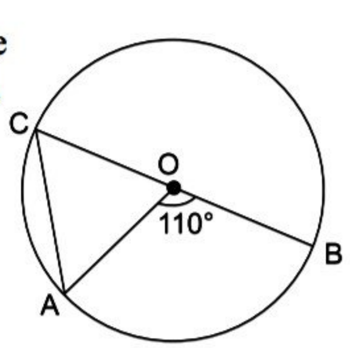
Now major arc AC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle.

$\therefore \quad \angle AOC = 2\angle ABC$

$\Rightarrow \quad \angle ABC = \frac{1}{2}\angle AOC = \frac{1}{2} \times 230° = 115°.$

3. From the figure, $\angle AOB = 110°$

(*i*) Now, arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle.



$\therefore \quad \angle AOB = 2\angle ACB$

$\Rightarrow \quad \angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 110° = 55°$

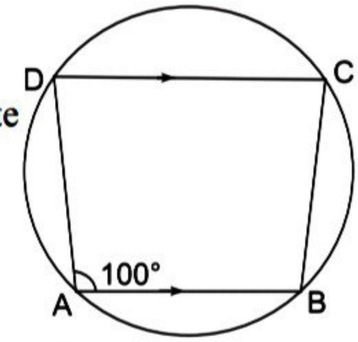
$\Rightarrow \quad \angle ACO = 55°.$

(*ii*) Now in ΔOAC, we have

$OA = OC$ [Radii of the same circle]

$\therefore \angle CAO = \angle ACO = 55^\circ$ [Angles opposite to equal sides are equal]

4. From the figure, ABCD is a cyclic quadrilateral.



$AB \parallel CD$ and $\angle BAD = 100^\circ$

(*i*) $\angle BAD + \angle BCD = 180^\circ$ [Sum of the opposite angles of a cyclic quadrilateral]

$\Rightarrow 100^\circ + \angle BCD = 180^\circ$

$\Rightarrow \angle BCD = 180^\circ - 100^\circ = 80^\circ$

(*ii*) $\because DC \parallel AB$

$\therefore \angle BAD + \angle ADC = 180^\circ$ [Sum of angles on the same side of a transversal]

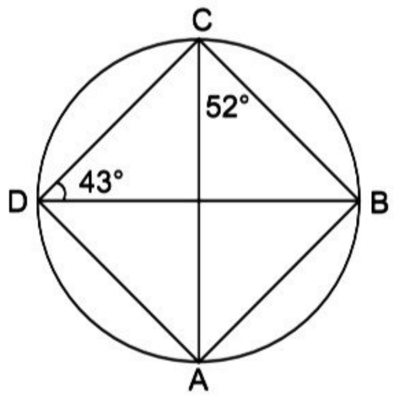
$\Rightarrow 100^\circ + \angle ADC = 180^\circ \Rightarrow \angle ADC = 180^\circ - 100^\circ = 80^\circ$

(*iii*) $\angle ABC + \angle ADC = 180^\circ$

[Sum of opposite angles of a cyclic quadrilateral]

$\Rightarrow \angle ABC + 80^\circ = 180^\circ \Rightarrow \angle ABC = 180^\circ - 80^\circ = 100^\circ$.

5. From the figure,



(*i*) $\angle ADB = \angle ACB = 52^\circ$ [Angles in the same segment]

[$\because \angle ACB = 52^\circ$]

(*ii*) $\angle BAC = \angle BDC = 43^\circ$ [Angles in the same segment]

[$\because \angle BDC = 43^\circ$]

(*iii*) In ΔABC, we have

$\angle ABC + \angle BCA + \angle BAC = 180^\circ$ [Sum of angles of a triangle]

$\Rightarrow \angle ABC + 52^\circ + 43^\circ = 180^\circ \Rightarrow \angle ABC + 95^\circ = 180^\circ$

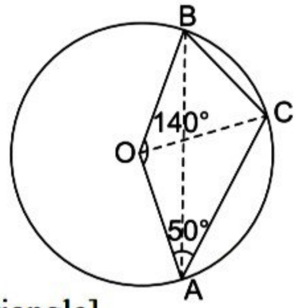
$\Rightarrow \angle ABC = 180^\circ - 95^\circ = 85^\circ$

Hence, $\angle ABC = 85^\circ$.

6. O is the centre of the circle

$\angle AOB = 140^\circ, \angle OAC = 50^\circ$

Join OC and AB.



In ΔOAC, we have:

$OA = OC$ [Radii of the same circle]

$\therefore \quad \angle OCA = \angle OAC = 50^\circ$ [$\because \angle OAC = 50^\circ$]

But in ΔAOC, we have

$\angle AOC + \angle OAC + \angle ACO = 180^\circ$ [Angle sum of a triangle]

$\Rightarrow \quad \angle AOC + 50^\circ + 50^\circ = 180^\circ$

$\Rightarrow \quad \angle AOC + 100^\circ = 180^\circ$

$\Rightarrow \quad \angle AOC = 180^\circ - 100^\circ = 80^\circ$

$\therefore \quad \angle BOC = 140^\circ - 80^\circ = 60^\circ$

(*i*) Now arc AC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle

$\therefore \ \angle AOC = 2\ \angle ABC = 180^\circ$

$\Rightarrow \angle ABC = \frac{1}{2}\ \angle AOC = \frac{1}{2} \times 80^\circ = 40^\circ$

(*ii*) In ΔOBC, we have $OB = OC$ [Radii of the same circle]

$\therefore \quad \angle OBC = \angle\ BCO$ [Angles opposite to equal sides are equal]

But, $\angle BOC + \angle OBC + \angle BCO = 180^\circ$

$60^\circ + \angle OBC + \angle BCO = 180^\circ$

$\Rightarrow 2\angle BCO + 180^\circ - 60^\circ = 120^\circ$

$\Rightarrow \quad \angle BOC = \frac{120''}{2} = 60^\circ \quad \Rightarrow \quad \angle BCO = 60^\circ$.

(*iii*) In ΔOAB, we have

$OB = OA$ [Radii of the same circle]

$\therefore \quad \angle OAB = \angle\ OBA$ [Angles opposite to equal sides]

But, $\angle AOB\ + \angle OAB\ + \angle OBA = 180^\circ$

$\Rightarrow 140^\circ + \angle OAB\ + \angle OBA = 180^\circ$

$\Rightarrow\ 140^\circ + 2\angle OBA = 180^\circ$

$\Rightarrow\ 2\angle OBA = 180^\circ - 140^\circ = 40^\circ$

$\therefore\ \angle OBA = \frac{40\Upsilon}{2} = 20^\circ \quad \Rightarrow \quad \angle OAB = 20^\circ$

(*iv*) $\angle BCA\ = \angle OCB\ + \angle ACO = 60^\circ + 50^\circ = 110^\circ$

7. From the figure, ABCD is a cyclic quadrilateral,

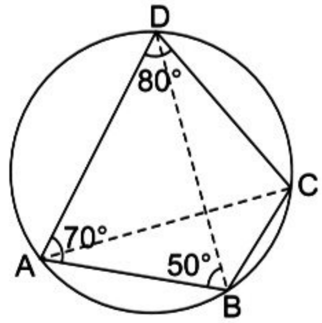
$\angle BAD = 70^\circ$, $\angle ABD = 50^\circ$ and $\angle ADC = 80^\circ$. Join AC.

(*i*) $\angle BDC = \angle ADC - \angle ADB = 80^\circ - \{180^\circ - (\angle DAB + \angle ABD)\}$

$\Rightarrow \quad \angle BDC = 80^\circ - \{180^\circ - (70^\circ + 50^\circ)\}$

$= 80^\circ - 180^\circ + 70^\circ + 50^\circ$

$\Rightarrow \quad \angle BDC = 200^\circ - 180^\circ = 20^\circ.$



(*ii*) $\angle BCD = 180^\circ - \angle BAD = 180^\circ - 70^\circ = 110^\circ$ [Since, ABCD is cyclic]

(*iii*) $\angle BCA = \angle ADB \Rightarrow \angle ADB = \angle ADC - \angle BDC$

$= 80^\circ - 20^\circ = 60^\circ$

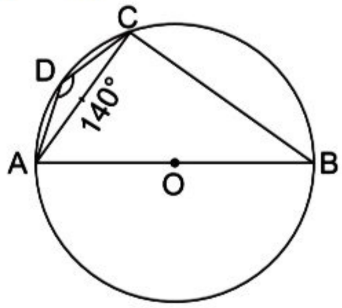
8. ABCD is a cyclic quadrilateral and AOB is the diameter of the circle.

Given that, $\angle ADC = 140^\circ$

$\angle ABC + \angle ADC = 180^\circ$ [Opposite angles of a cyclic quadrilateral]

$\Rightarrow \angle ABC + 140^\circ = 180^\circ \Rightarrow \angle ABC = 180^\circ - 140^\circ = 40^\circ$

Now in ΔABC, we have



$\angle ACB = 90^\circ$ [Angle in a semi –circle]

$\angle ABC = 40^\circ$ [Proved]

But $\angle BAC + \angle ACB + \angle ABC = 180^\circ$

[Angle of the triangle]

$\Rightarrow \angle BAC + 90^\circ + 40^\circ = 180^\circ$

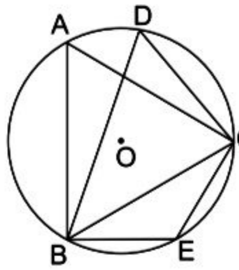
$\Rightarrow \angle BAC + 130^\circ = 180^\circ \Rightarrow \angle BAC = 180^\circ - 130^\circ = 50^\circ$

9. From the figure, O is the centre of the circle and Δ ABC is an equilateral triangle.

(*i*) $\angle BAC = \angle ABC = \angle ACB = 60^\circ.$

$\angle BAC = \angle BDC$ [Angle in the same segment]

$\therefore \angle BDC = 60^\circ$



(*ii*) $\because$ ABEC is a cyclic quadrilateral .

$\therefore \angle A + \angle BEC = 180^\circ \Rightarrow 60^\circ + \angle BEC = 180^\circ$

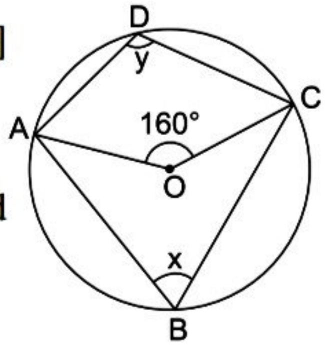
$\Rightarrow \angle BEC = 180^\circ - 60^\circ = 120^\circ$

10. Given O is the centre of the circle $\angle AOC = 160^\circ$, $\angle ABC = x$ and $\angle ADC = y$

To Prove. $3\angle y - 2\angle x = 140^\circ$

Proof: $\therefore \angle AOC + \text{reflex } \angle AOC = 360^\circ$

[Angles at a point]



$\Rightarrow \quad 160^\circ + \text{Reflex } \angle AOC = 360^\circ$

$\Rightarrow \quad \text{Reflex } \angle AOC = 360^\circ - 160^\circ = 200^\circ$

Now arc ADC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle.

$\therefore \quad \angle AOC = 2x \Rightarrow 2x = 160^\circ$

$$\Rightarrow \qquad x = \frac{160''}{2} = 80^\circ$$

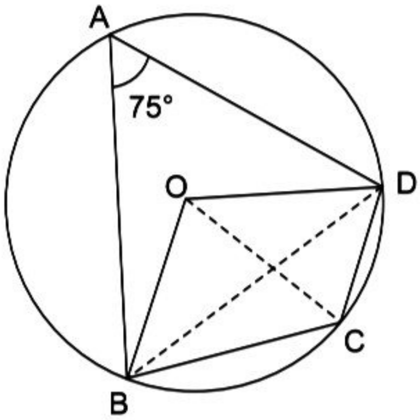
Similarly, reflex $\angle AOC = 2y$

$$\Rightarrow \qquad 2y = 200^\circ \quad \Rightarrow \quad y = \frac{200''}{2} = 100^\circ$$

Now , L.H.S $= 3\angle y - 2\angle x = 3 \times 100^\circ - 2 \times 80^\circ$

$= 300^\circ - 160^\circ = 140^\circ =$ R.H.S,

11. (*i*) From figure , O is the centre of the circle, $\angle BAD = 75^\circ$, chord BC = chord CD. Join BD, OC. arc BCD subtends $\angle BOD$ at the centre and $\angle BAD$ at the remaining part.



$\Rightarrow \quad \angle BOD = 2\angle BAD = 2 \times 75^\circ = 150^\circ$ ...(*i*)

But $\quad BC = CD$. So, $\angle BOC = \angle COD$.

[Equal chords subtend equal angles at the centre]

So, $\quad \angle BOD = \angle BOC + \angle COD = \angle BOC + \angle BOC$

$= 2\,\angle BOC = 150^\circ$

$$\Rightarrow \angle BOC = \frac{150}{2} = 75^\circ$$

(*ii*) $\angle OBD = \frac{1}{2}\,[\,180^\circ - \angle BOD] = \frac{1}{2}[180^\circ - 150^\circ] = \frac{1}{2}\,[30^\circ] = 15^\circ$

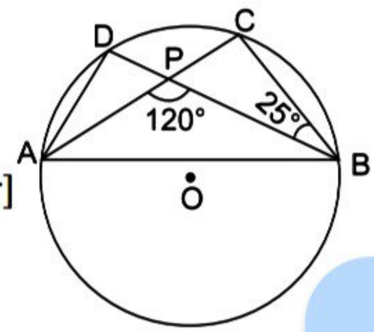
(*iii*) $\angle BCD + \angle BAD = 180^\circ \Rightarrow \angle BCD + 75^\circ = 180^\circ$

$\Rightarrow \angle BCD = 180^\circ - 75^\circ = 105^\circ$.

12. Given: O is the centre of the circle.

$\angle CBD = 25^\circ, \angle APB = 120^\circ$

To find : $\angle ADB$



Proof: In ΔABC, we have

$\angle BPC = 180^\circ - \angle APB \quad [\because$ Linear pair]

$= 180^\circ - 120^\circ = 60^\circ$

In ΔPCB, we have

$\angle BPC + \angle CBP + \angle PCB = 180^\circ$ [Angle sum of a triangle]

$\Rightarrow 60^\circ + 25^\circ + \angle PCB = 180^\circ$

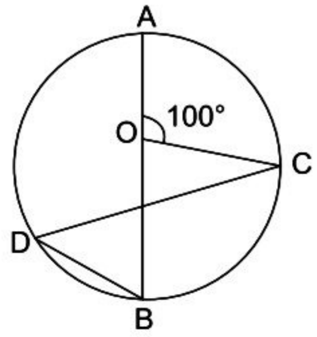
$\angle PCB = 180^\circ - 60^\circ - 25^\circ$

$= 180^\circ - 85^\circ = 95^\circ$

$\angle ADB = \angle PCB = 95^\circ$ [Angles in the same segment]

13. From the figure AOB is the diameter of the circle with centre O, $\angle AOC = 100^\circ$.

But, $\angle AOC + \angle BOC = 180^\circ$ [A linear pair]



$\Rightarrow 100^\circ + \angle BOC = 180^\circ$

$\Rightarrow \angle BOC = 180^\circ - 100^\circ = 80^\circ$

Now arc BC subtends $\angle BOC$ at the centre and $\angle BDC$ at the remaining part of the circle.

$\therefore \angle BOC = 2\angle BDC$

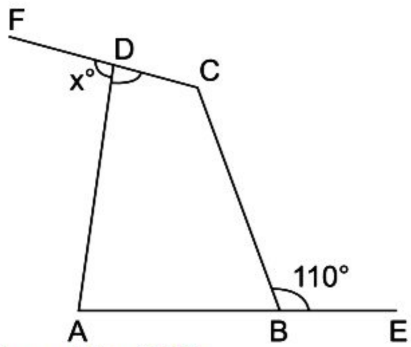
$\Rightarrow \angle BDC = \frac{1}{2}\angle BOC$

$\Rightarrow \angle BDC = \frac{1}{2} \times 80^\circ = 40^\circ$

Hence, $\angle BDC = 40^\circ$.

14. When $x^\circ = 70^\circ$, then $\angle ADC = 180^\circ - x$

$= 180^\circ - 70^\circ = 110^\circ$



$\therefore \angle ADC = \angle CBE = 110^\circ$

$\therefore \angle ABC = 180^\circ - 110^\circ = 70^\circ$

$\therefore \angle ABC + \angle ADC = 70^\circ + 110^\circ = 180^\circ$

So, the sum of the opposite angles of a quadrilateral is 180°.

$\therefore$ ABCD is a cyclic quadrilateral.

Hence, A,B,C, and D are concyclic.

(*ii*) When $x^\circ = 80^\circ$ then $\angle ADC = 180^\circ - x$

$= 180^\circ - 80^\circ = 100^\circ$

And $\angle ABC = 180^\circ - 110^\circ = 70^\circ$

$\therefore \angle ADC + \angle ABC = 100^\circ + 70^\circ = 170^\circ$

$\because$ Sum of the opposite angles of a quadrilateral is not equal to 180°.

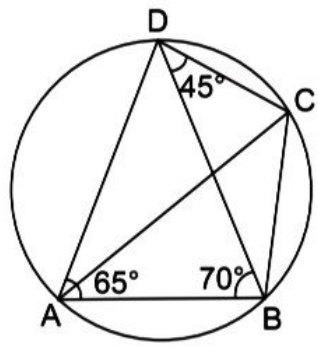
$\therefore$ ABCD is not a cyclic quadrilateral.

Hence, A,B,C, and D are not concyclic.

15. (*i*) ABCD is a cyclic quadrilateral.

$\therefore$ $\angle BAD + \angle BCD = 180°$

[Opposite angles of a cyclic quadrilateral are supplementary]



$\Rightarrow \angle 65° + \angle BCD = 180°$

$\Rightarrow \angle BCD = 180° - 65° = 115°$

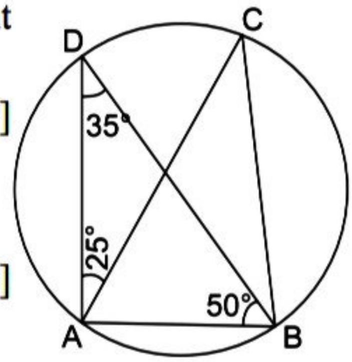
(*ii*) In ΔABD, $\angle BAD + \angle ABD + \angle ADB = 180°$

$\Rightarrow$ $65° + 70° + \angle ADB = 180°$ $\Rightarrow$ $\angle ADB = 45°$

$\angle ADC = \angle ADB + \angle BDC = 45° + 45° = 90°$, hence AC is a diameter.

16. From the figure,

$\angle CAD = 25°$, $\angle ABD = 50°$ and $\angle ADB = 35°$



(*i*) $\angle CBD$ and $\angle CAD$ are in the same segment of a circle.

$\therefore$ $\angle CBD = \angle CAD = 25°$ [$\because \angle CAD = 25°$]

(*ii*) In ΔABD, we have

$\angle ADB + \angle ABD + \angle DAB = 180°$

[Sum of angles of a triangle]

$\Rightarrow$ $35° + 50° + \angle DAB = 180°$

$\Rightarrow$ $85° + \angle DAB = 180°$

$\Rightarrow$ $\angle DAB = 180° - 85° = 95°$

$\Rightarrow$ $\angle CAB + \angle DAC = 95°$

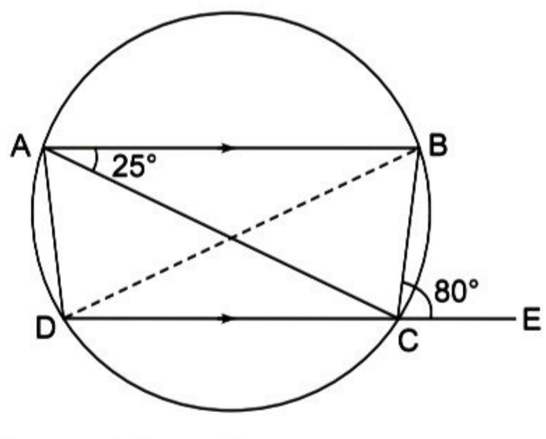
$\Rightarrow$ $\angle CAB + 25° = 95°$

$\therefore$ $\angle CAB = 95° - 25° = 70°$

(*iii*) $\angle ADB$ and $\angle ACB$ are in the same segment.

$\therefore$ $\angle ACB = \angle ADB = 35°$ [$\because \angle ADB = 35°$]

17. From the figure, AB || DC and $\angle BCE = 80°$ and $\angle BAC = 25°$. Join BD.



ABCD is a cyclic quadrilateral.

(*i*) $\angle BAD = \angle BCE = 80°$ [Ext. angle of a cyclic quadrilateral is equal to its interior opposite angle]

$\Rightarrow \angle BAC + \angle CAD = 80^\circ$

$\Rightarrow 25^\circ + \angle CAD = 80^\circ$

$\Rightarrow \angle CAD = 80^\circ - 25^\circ \Rightarrow \angle CAD = 55^\circ$

(*ii*) $\angle CBD = \angle CAD = 55^\circ$ [Angles in the same segment given]

(*iii*) $\because$ AB || DC and AD is its transversal [Given]

$\therefore \angle BAD + \angle ADC = 180^\circ$ [Co-interior angles]

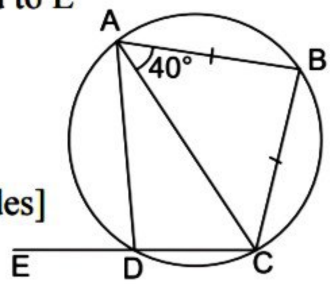
$\Rightarrow 80^\circ + \angle ADC = 180^\circ$

$\Rightarrow \angle ADC = 180^\circ - 80^\circ = 100^\circ$

18. Given:

ABCD is a cyclic quadrilateral side CD is produced to E

BA = BC and $\angle BAC = 40^\circ$



To find : $\angle ADE$

Proof: In ΔABC, AB = BC

$\therefore \angle BAC = \angle BCA$ [Angles opposite to equal sides]

$\therefore \angle BCA = 40^\circ$

And $\angle ABC = 180^\circ - (40^\circ + 40^\circ)$

$= 180^\circ - 80^\circ = 100^\circ$

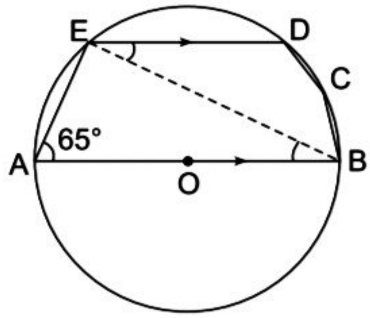
But in cyclic quad. ABCD

Ext. $\angle ADE = \angle ABC$ [Interior opposite angle]

$\therefore \angle ADE = 100^\circ$.

19. From the figure,

AOB is the diameter of the circle with centre O. Chord ED || AB and $\angle EAB = 65^\circ$. Join EB.



(*i*) In ΔAEB, we have

$\angle AEB + \angle EAB + \angle EBA = 180^\circ$

$\Rightarrow 90^\circ + 65^\circ + \angle EBA = 180^\circ$

$\Rightarrow 155^\circ + \angle EBA = 180^\circ$

$\Rightarrow \angle EBA = 180^\circ - 155^\circ = 25^\circ$

$\therefore \angle EBA = 25^\circ$

(*ii*) $\because$ ED || AB

$\therefore \angle EAB + \angle AED = 180^\circ$ [Angles on the same side of the transversal]

$\Rightarrow 65^\circ + \angle AED = 180^\circ$

$\Rightarrow \angle AED = 180^\circ - 65^\circ = 115^\circ$

$\therefore \angle BED = \angle AED - \angle AEB = 115^\circ - 90^\circ = 25^\circ$.

(*iii*) $\because$ EBCD is a cyclic quadrilateral.

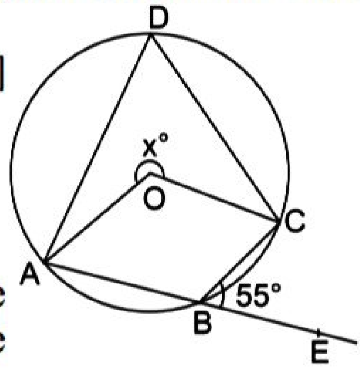
$\therefore \angle BCD + \angle BED = 180^\circ \Rightarrow \angle BCD + 25^\circ = 180^\circ$

$\Rightarrow \angle BCD = 180^\circ - 25^\circ$

$\therefore \angle BCD = 155^\circ$

20. From the figure, we have

O is the centre of the circle. ABCD is cyclic quadrilateral . ABE is a straight line and $\angle CBE = 55^\circ$



$\angle ABC + \angle CBE = 180^\circ$ [Linear pair]

$\Rightarrow \angle ABC + 55^\circ = 180^\circ$

$\Rightarrow \angle ABC = 180^\circ - 55^\circ$

$\Rightarrow \angle ABC = 125^\circ$

Now major arc ADC subtends reflex $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle

$\therefore\ x = 2 \times 125^\circ = 250^\circ$

In cyclic quadrilateral ABCD, we have

$\angle ADC + \angle ABC = 180^\circ$

$\Rightarrow \angle ADC + 125^\circ = 180^\circ$

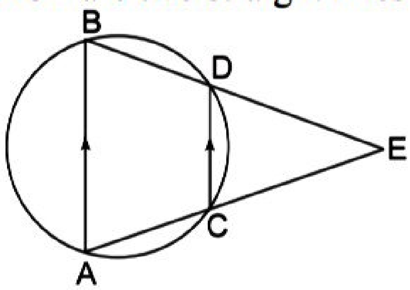
$\Rightarrow \angle ADC = 180^\circ - 125^\circ$

$\therefore\ \angle ADC = 55^\circ$

Hence, (i) $\angle ADC = 55^\circ$ (ii) $\angle ABC = 125^\circ$ (iii) $x = 250^\circ$.

21. **Given:**

AB and CD, are two parallel chords. BDE and ACE are two straight lines intersecting each other at E outside the circle.



To prove: ΔAEB, is an isosceles triangle.

Proof . ABCD is a cyclic quadrilateral

$\therefore$ Ext. $\angle EDC = \angle A$ and

Ext. $\angle DCE = \angle B$

But AB || CD

$\Rightarrow \angle EDC = \angle B$ [Corresponding angles]

and $\angle DCE = \angle A \Rightarrow \angle B = \angle A$

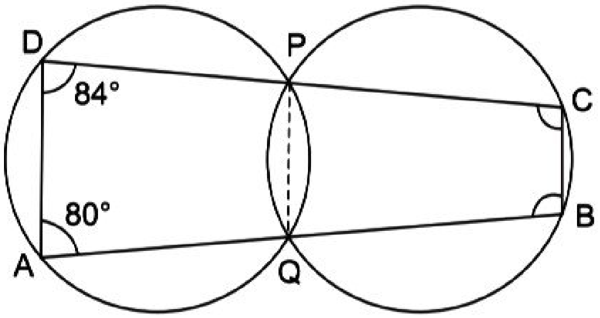
$\therefore$ EA = EB

Hence, $\Delta AEB$is an isosceles triangle.

22. From the figure, two circles intersect each other at P and Q

ABCD is a quadrilateral in which $\angle A = 80^\circ$ and $\angle D = 84^\circ$. Join PQ.

AQPD is a cyclic quadrilateral.



$\therefore\ \angle ADP + \angle AQP = 180^\circ$

$\Rightarrow 84^\circ + \angle AQP \ = 180^\circ$

$\Rightarrow \angle AQP = 180^\circ - 84^\circ = 96^\circ$

Similarly, $\angle QAD + \angle QPD = 180^\circ$

$\Rightarrow 80^\circ + \ \angle QPD \ = 180^\circ$

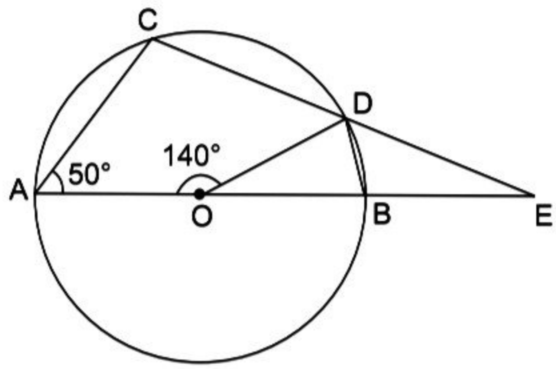
$\Rightarrow \angle QPD = 180^\circ - 80^\circ = 100^\circ$

Now , in cyclic quadrilateral QBCP,

Ext, $\angle QPD = \angle QBC$

$\therefore$ (i) $\angle QBC = \angle QPD = 100^\circ$ (ii) $\angle BCP = \angle AQP = 96^\circ$.

23. From the figure, O is the centre of the circle.



$\angle AOD = 140^\circ$ and $\angle CAB = 50^\circ$

$\angle AOD + \angle DOB = 180^\circ$ [Linear pair]

$\Rightarrow 140^\circ + \angle DOB = 180^\circ$

$\Rightarrow \angle DOB = 180^\circ - 140^\circ = 40^\circ$

But, OB = OD [Radii of the same circle]

$\therefore\ \angle OBD = \angle ODB$ [Angles opposite to equal sides]

But in ΔOBD, we have

$$\angle OBD + \angle ODB + \angle BOD = 180^\circ$$

$\Rightarrow \angle OBD + \angle OBD + 40^\circ = 180^\circ$ [$\therefore \angle OBD = \angle ODB$]

$\Rightarrow 2\angle OBD = 180^\circ - 40^\circ = 140^\circ$

$\therefore \angle OBD = \dfrac{140''}{2} = 70^\circ$

(*i*) In cyclic quadrilateral ABCD,

Ext. $\angle EDB = \angle CAB = 50^\circ$ [$\because \angle CAB = 50^\circ$]

(*ii*) $\angle EBD + \angle OBD = 180^\circ$ [Linear pair]

$\Rightarrow \angle EBD + 70^\circ \quad = 180^\circ$

$\Rightarrow \angle EBD = 180^\circ - 70^\circ = 110^\circ$

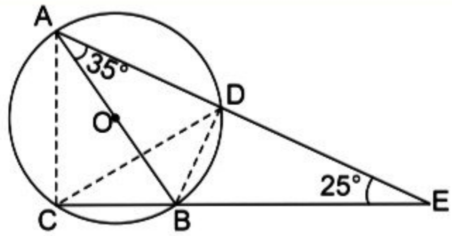
$\therefore\ \angle EBD = 110^\circ$.

Join BD, CA and CD.

In ΔABD, we have

$\because \quad \angle ADB = 90^\circ$ [Angle in semi-circle]

$\therefore \quad \angle BDE = 180^\circ - 90^\circ = 90^\circ$



In ΔBED, we have $\angle DBE = 180^\circ - (90^\circ + 25^\circ) = 180^\circ - 115^\circ = 65^\circ$

But $\quad \angle CBD + \angle DBE = 180^\circ \Rightarrow \angle CBD + 65^\circ = 180^\circ$

$\Rightarrow \angle CBD = 180^\circ - 65^\circ = 115^\circ$

$\because \angle BCD = \angle BAD$[Angles in the same segment]

$\therefore \angle BCD = 35^\circ \qquad (\because \angle BAD = 35^\circ)$

Now, In ΔCBD, we have $\angle DCB + \angle DBC + \angle BDC = 180^\circ$

$\Rightarrow 35^\circ + 115^\circ + \angle BDC = 180^\circ$

$\Rightarrow 150^\circ + \angle BDC = 180^\circ$

$\Rightarrow \angle BDC = 180^\circ - 150^\circ$

$\Rightarrow \angle BDC = 30^\circ$.

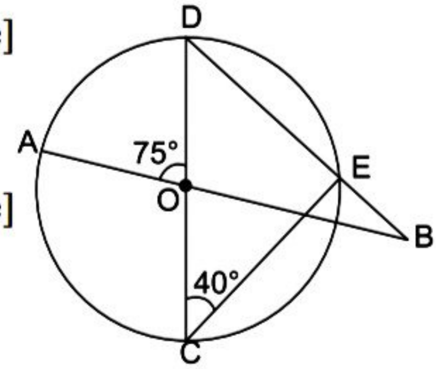
Hence (*i*) $\angle DCB = 35^\circ$

(*ii*) $\angle DBC = 115^\circ$ and

(*iii*) $\angle BDC = 30^\circ$

25. From the figure, lines AB and CD pass through the centre O of the circle. $\angle AOD = 75^\circ$ and $\angle OCE = 40^\circ$

(*i*) $\quad \angle CED = 90^\circ$ [Angle in a semi-circle]



Now, in ΔCDE, we have

$\angle CDE + \angle CED + \angle ECD = 180^\circ$

[Angle sum of a triangle]

$\Rightarrow \angle CDE + 90^\circ + 40^\circ = 180^\circ$

$\Rightarrow \angle CDE + 130^\circ = 180^\circ$

$\Rightarrow \angle CDE = 180^\circ - 130^\circ$

$\Rightarrow \angle CDE = 50^\circ$

(*ii*) Now, in ΔOBD, we have

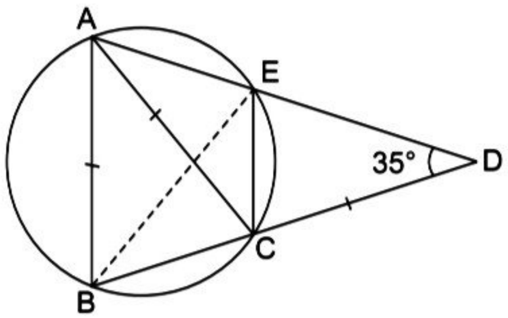
Ext. $\angle DOA = \angle CDE + \angle OBD$

$\Rightarrow 75^\circ = 50^\circ + \angle OBD$

$\Rightarrow \angle OBD = 75^\circ - 50^\circ = 25^\circ$.

$\Rightarrow \angle OBE = 25^\circ$.

26. From the figure,



$$AB = AC = CD, \angle ADC = 35°$$

$\because$ $AC = DC$

$\therefore \angle CAD = \angle ADC = 35°$

Now, in ΔCDA, we have

(*i*) Ext. $\angle ACB = \angle CAD + \angle ADC$

$= 35° + 35° = 70°$

$\because$ $AB = AC$

$\therefore$ $\angle ABC = \angle ACB = 70°$

(*ii*) Now, in ΔABC, we have

$\angle ABC + \angle ACB + \angle BAC = 180°$ [Sum of angles of a triangle]

$\Rightarrow 70° + 70° + \angle BAC = 180°$

$\Rightarrow 140° + \angle BAC = 180°$

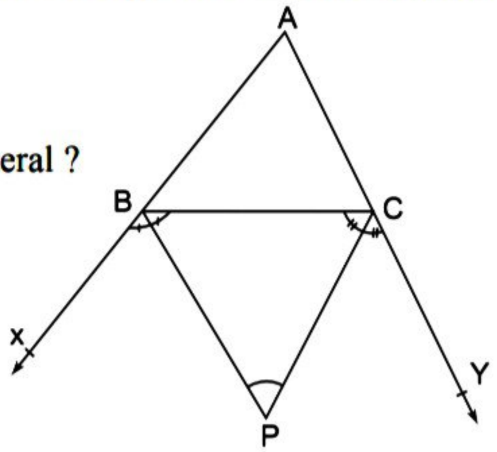
$\Rightarrow \angle BAC = 180° - 140°$

$\therefore \angle BAC = 40°$

But $\angle BAC = \angle BEC$ [Angles in the same segment]

$\therefore \angle BEC = 40°$.

27. Given: The sides AB and AC of a ΔABC, are produced to X and Y respectively. BP and CP are the bisectors of Ext $\angle B$ and Ext $\angle C$ meeting each other at P.



To Prove: (*i*) $\angle BPC = 90° - \frac{\angle A}{2}$

(*ii*) Is ABPC a cyclic quadrilateral ?

Proof.: In ΔABC

Ext. $\angle B$ = Interior $\angle C + \angle A$

Ext. $\angle C$ = Interior $\angle B + \angle A$

or $\angle CBP = \frac{1}{2}(\angle C + \angle A)$

$= \frac{1}{2}\angle C + \frac{1}{2}\angle A$

and $\angle BCP = \frac{1}{2}(\angle B + \angle A) = \frac{1}{2}\angle B + \frac{1}{2}\angle A$

On adding , We get :

$\angle CBP + \angle BCP = \frac{1}{2}\angle C + \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2} A$

$= \frac{1}{2}(\angle A + \angle B + \angle C) + \frac{1}{2}\angle A$

$$= \frac{1}{2} \times 180° + \frac{1}{2}\angle A = 90° + \frac{1}{2}\angle A$$

But in ΔBPC , we have

$\angle BPC = 180° - (CBP + \angle BCP)$

$$= 180° - \left[90" + \frac{1}{2}\angle A\right]$$

$$= 180° - 90° - \frac{1}{2}\angle A = 90° - \frac{1}{2}\angle A$$

(*ii*) In quadrilateral ABPC , we have

$$\angle A + \angle BPC = \angle A + 90° - \frac{1}{2}\angle A$$

$$= 90° + \frac{1}{2}\angle A$$

But it is not equal to 180°

$\therefore$ ABPC is not a cyclic quadrilateral.

28. I is the incentre of the ΔABC , AI is joined and produced to meet the circle at D. DB, DC, IC, and IB are joined. $\angle ABC = 55°$ and $\angle ACB = 65°$

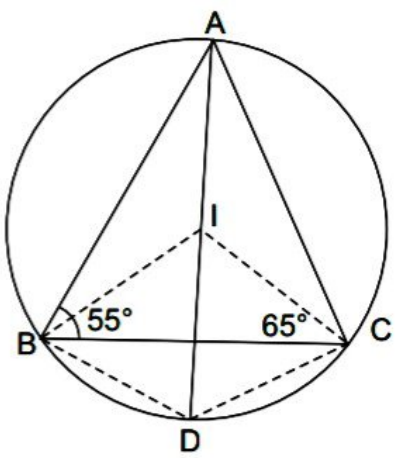
(*i*) $\because$ AD is the diameter

$\therefore \angle ACD = 90°$ [Angle in a semi – circle]

$\Rightarrow \angle ACB + \angle BCD = 90°$

$\Rightarrow 65° + \angle BCD = 90°$

$\Rightarrow \angle BCD = 90° - 65° = 25°$



(*ii*) Similarly, $\angle ABD = 90°$

$\Rightarrow \angle ABC + \angle CBD = 90°$

$\Rightarrow 55° + \angle CBD = 90°$

$\Rightarrow \angle CBD = 90° - 55° = 35°$

(*iii*) In ΔABC , we have

$\angle BAC + \angle ABC + \angle ACB = 180°$

$\Rightarrow \angle BAC + 55° + 65° = 180°$ [Angle sum of triangle]

$\Rightarrow \angle BAC + 120° = 180°$

$\Rightarrow \angle BAC = 180° - 120°$

$\angle BAC = 60°$

$\because$ I is the incenter of ΔABC.

$\therefore$ I lines on the bisector of $\angle BAC$

$$\therefore \angle BAI = \angle CAI = \frac{60Υ}{2} = 30°$$

So, $\angle BAD = \angle CAD = 30^\circ$

$\because$ I line on the angle bisector of $\angle ACB$

$\therefore \angle ACI = \frac{65\Upsilon}{2} = 32\frac{1\Upsilon}{2} = 32.5^\circ$

Now, $\angle DCI = \angle ACD - ACI$

$$= 90^\circ - 32\frac{1\Upsilon}{2} = 57\frac{1\Upsilon}{2} = 57.5^\circ$$

(*iv*) $\because$ BI is the angle bisector of $\angle ABC$

$$\angle IBA = \angle IBC = \frac{55\Upsilon}{2} = 27.5^\circ$$

Now, in ΔBIC, we have

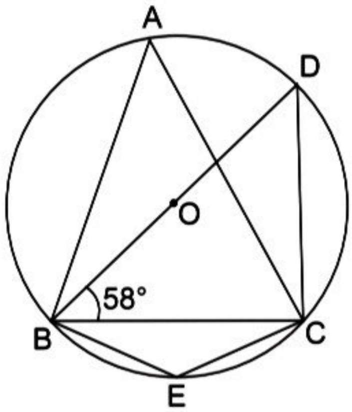
$\angle BIC + \angle ICB + \angle IBC = 180^\circ$ [Angle sum of a triangle]

$\Rightarrow \angle BIC + (32.5^\circ + 27.5^\circ) = 180^\circ$

$\Rightarrow \angle BIC + 60^\circ = 180^\circ$

$\Rightarrow \angle BIC = 180^\circ - 60^\circ = 120^\circ$

29. In the given figure, BD is the diameter of the circle, $\angle DBC = 58^\circ$.



Calculate

(*i*) $\angle BDC$ (*ii*) $\angle BEC$ (*iii*) $\angle BAC$

(*i*) In ΔBCD, we have

$\angle DBC = 58^\circ$, $\angle BCD = 90^\circ$ [Angle in a semicircle]

$\therefore \angle BDC = 180^\circ - (58^\circ + 90^\circ)$

$= 180^\circ - 148^\circ = 32^\circ$

(*ii*) BECD is a cyclic quadrilateral

$\therefore \angle BEC + \angle BDC = 180^\circ$ [Sum of opposite angles of a quadrilateral]

$\Rightarrow \angle BEC + 32^\circ = 180^\circ$

$\Rightarrow \angle BEC = 180^\circ - 32^\circ \Rightarrow \angle BEC = 148^\circ$

(*iii*) $\angle BAC = \angle BDC = 32^\circ$ [Angles in the same segment]