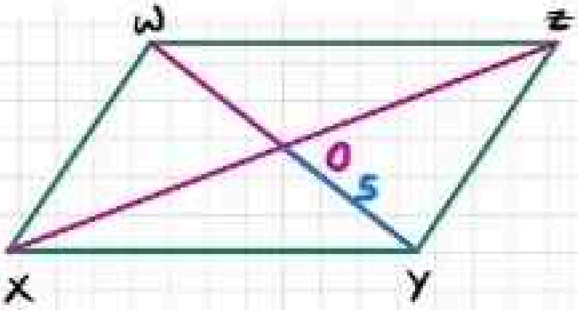
1. Diagonals of a parallelogram WXYZ intersect each other at point O. If $\angle XYZ = 135°$ then what is the measure of $\angle XWZ$ and $\angle YZW$?
If $l(OY) = 5$ cm then $l(WY) = $?



GIVEN : WY and XZ intersect at O, $\angle XYZ = 135°$;
$l(OY) = 5$ cm

FIND : $m\angle XWZ$, $m\angle YZW$, $l(WY)$

SOLUTION:

In a parallelogram, opposite angles are congruent

$\therefore \angle XWZ = \angle XYZ = 135°$ $\quad \therefore \boxed{m\angle XWZ = 135°}$

Also, adjacent angles of a parallelogram are supplementary

$\therefore m\angle XYZ + m\angle YZW = 180°$

$\therefore 135° + m\angle YZW = 180°$

$\therefore m\angle YZW = 180° - 135°$

$\therefore \boxed{m\angle YZW = 45°}$

Diagonals of a parallelogram bisect each other

$\therefore l(OY) = \frac{1}{2} l(WY)$

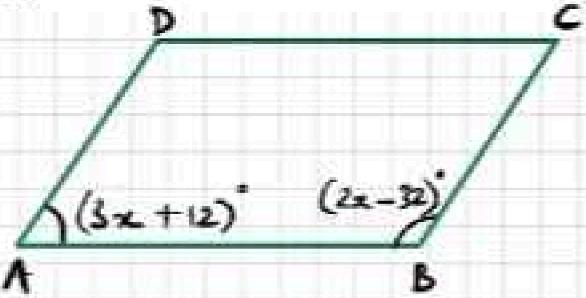
$\therefore 5 = \frac{1}{2} l(WY)$

$\therefore 5 \times 2 = l(WY)$

$\therefore 10 = l(WY)$

$\therefore \boxed{l(WY) = 10 \text{ cm}}$

2. In a parallelogram ABCD, If $\angle A = (3x + 12)^\circ$, $\angle B = (2x - 32)^\circ$ then find the value of x and then find the measures of $\angle C$ and $\angle D$.



GIVEN: $\angle A = (3x + 12)^\circ$

$\angle B = (2x - 32)^\circ$

FIND: x, $m\angle C$, $m\angle D$

SOLUTION:

Adjacent angles of a parallelogram are supplementary

$\therefore m\angle A + m\angle B = 180^\circ$

$\therefore (3x + 12)^\circ + (2x - 32)^\circ = 180^\circ$

$\therefore 3x + 12 + 2x - 32 = 180$

$\therefore 5x - 20 = 180$

$\therefore 5x = 180 + 20$

$\therefore 5x = 200$

$\therefore x = \frac{\cancel{200}^{40}}{\cancel{5}_{1}}$ $\therefore$ $\boxed{x = 40}$

Now, $m\angle A = [3x + 12]^\circ$

$= [3(40) + 12]^\circ$

$= [120 + 12]^\circ$

$= 132^\circ$

$m\angle B = [2x - 32]^\circ$

$= [2(40) - 32]^\circ$

$= [80 - 32]^\circ$

$= 48^\circ$

In a parallelogram, opposite angles are congruent

$\therefore m\angle C = m\angle A = 132^\circ$

& $m\angle D = m\angle B = 48^\circ$

$\therefore$ $\boxed{m\angle C = 132^\circ \quad m\angle D = 48^\circ}$

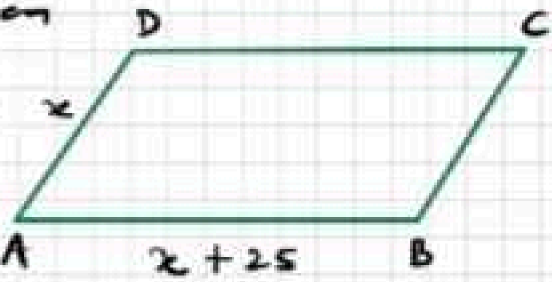
Ans. The value of x is 40 and the measures of $\angle C$ & $\angle D$ are 132° and 48°

3. Perimeter of a parallelogram is 150 cm. One of its sides is greater than the other side by 25 cm. Find the lengths of all sides.

GIVEN: Perimeter of parallelogram is 150 cm

One side greater than other by 25cm

FIND: lengths of all sides



SOLUTION:

Let in parallelogram ABCD, $l(AD) = x$ cm

One of its sides is greater than the other side by 25 cm

$\therefore AB = (x + 25)$ cm

In a parallelogram, opposite sides are congruent

$\therefore AB = CD = (x + 25)$ cm

& $AD = BC = x$ cm

Now, Perimeter of $\square ABCD = 150$ cm ...(given)

$\therefore AB + BC + CD + AD = 150$

$\therefore (x+25) + x + (x+25) + x = 150$

$\therefore x + 25 + x + x + 25 + x = 150$

$\therefore 4x + 50 = 150$

$\therefore 4x = 150 - 50$

$\therefore 4x = 100$

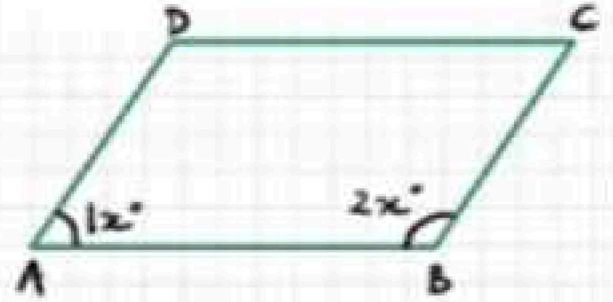
$\therefore x = \frac{\cancel{100}^{25}}{\cancel{4}_{1}}$ $\therefore$ $\boxed{x = 25}$

$\therefore AB = CD = (x + 25)$ cm $= (25+25)$ cm $= 50$cm

& $AD = BC = x$ cm $= 25$cm

<u>Ans</u>. The lengths of sides of the parallelogram

4. If the ratio of measures of two adjacent angles of a parallelogram is 1 : 2, find the measures of all angles of the parallelogram.



GIVEN: Ratio of measures of 2 adjacent angles of a parallelogram is 1:2

FIND: Measures of all angles

SOLUTION:

Let $\square ABCD$ be the parallelogram.

The ratio of measures of adjacent angles of a parallelogram is 1:2

Let the common multiple be x

$\therefore m\angle A = 1x°$ & $m\angle B = 2x°$

Now, $m\angle A + m\angle B = 180°$ ... (Adjacent angles of a parallelogram are supplementary)

$\therefore 1x + 2x = 180$

$\therefore 3x = 180$

$\therefore x = \frac{180}{3}$

$\therefore \boxed{x = 60}$

$\therefore m\angle A = x° = 60°$

$m\angle B = 2x° = 2 \times 60 = 120°$

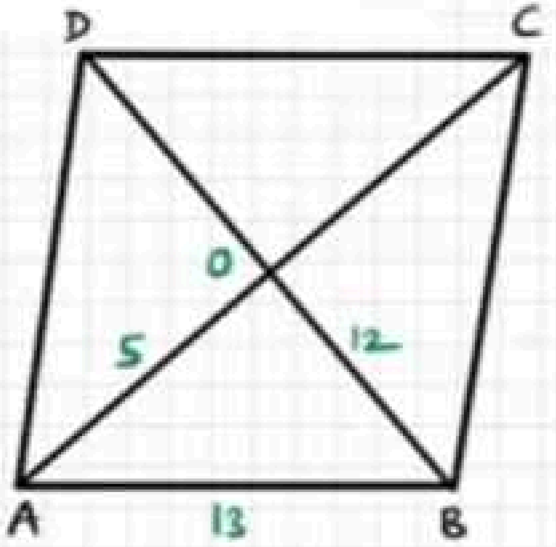
$m\angle C = m\angle A = 60°$

$m\angle D = m\angle B = 120°$

... (Opposite angles of a parallelogram are congruent)

Ans. The measures of the angles of the parallelogram are 60°, 120°, 60° and 120°.

5'. Diagonals of a parallelogram intersect each other at point O. If AO = 5, BO = 12 and AB = 13 then show that □ABCD is a rhombus.



GIVEN: In parallelogram ABCD

AO = 5 , BO = 12, AB = 13

TO PROVE: □ABCD is a rhombus

($\angle AOB = 90°$)

PROOF:

In $\triangle AOB$,

AO = 5 , BO = 12 , AB = 13 ..(Given)

Also,

$AO^2 + BO^2$	AB^2
$= 5^2 + 12^2$	$= 13^2$
$= 25 + 144$	$= 169$ ——— (2)
$= 169$ ——— (1)	

$\therefore AB^2 = AO^2 + BO^2$ ... (From 1 & 2)

$\therefore \triangle AOB$ is a right-angled triangle ... (Converse of Pythagoras Theorem)

$\therefore \angle AOB = 90°$

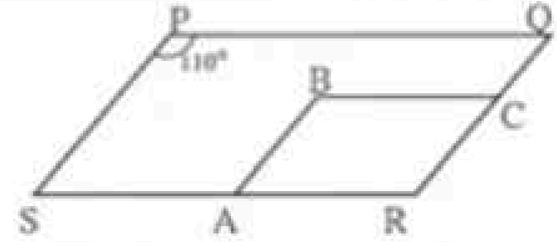
$\therefore AC \perp BD$

$\therefore$ Diagonals of parallelogram □ABCD are intersecting each other at right angles

$\therefore$ □ABCD is a rhombus

Hence, proved.

6. In the figure 5.12, $\square$PQRS and $\square$ABCR are two parallelograms. If $\angle P = 110°$ then find the measures of all angles of $\square$ABCR.



GIVEN : $\square$PQRS & $\square$ABCR are parallelograms

$\angle P = 110°$

FIND : $m\angle A$, $m\angle B$, $m\angle C$, $m\angle R$

SOLUTION:

$\square$PQRS is a parallelogram ... (GIVEN)

$\therefore m\angle R = m\angle P = 110°$ ... (Opposite angles of parallelogram)

In parallelogram $\square$ABCR

$m\angle B = m\angle R = 110°$ ... (Opposite angles of parallelogram)

Also, $m\angle B + m\angle C = 180°$ ... (Adjacent angles of a parallelogram are supplementary)

$\therefore 110° + m\angle C = 180°$

$\therefore m\angle C = 180° - 110°$

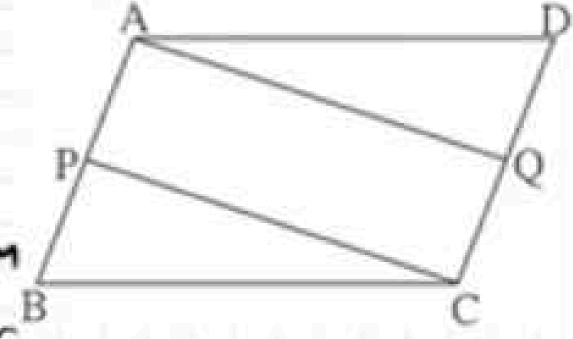
$m\angle C = 70°$

Now, $m\angle A = m\angle C = 70°$ ... (Opposite angles of parallelogram)

$\therefore$

$m\angle A = 70°$	$m\angle B = 110°$
$m\angle C = 70°$	$m\angle R = 110°$

1. In figure 5.22, □ABCD is a parallelogram, P and Q are midpoints of side AB and DC respectively, then prove □APCQ is a parallelogram.



GIVEN : □ABCD is a parallelogram
P & Q are midpoints of side AB & DC

TO PROVE : □APCQ is a parallelogram

PROOF :

Now, AB = DC ...(Opposite sides of parallelogram)

$\therefore$ $\frac{1}{2}$AB = $\frac{1}{2}$DC ...(Multiplying both sides by $\frac{1}{2}$)

AP = QC —(1) ... (P & Q are midpoints of AB & DC respectively)

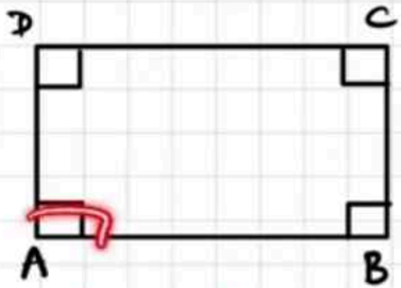
Also, AB ∥ DC ... (Opposite sides of a parallelogram)

$\therefore$ AP ∥ QC —(2) ... (A – P – B & D – Q – C)

$\therefore$ AP = QC and AP ∥ QC ... (From 1 & 2)

$\therefore$ □APCQ is a parallelogram ... [A quadrilateral is a parallelogram if a pair of its opposite sides are parallel & congruent]

2. Using opposite angles test for parallelogram, prove that every rectangle is a parallelogram.



GIVEN: □ABCD is a rectangle

TO PROVE: □ABCD is a parallelogram

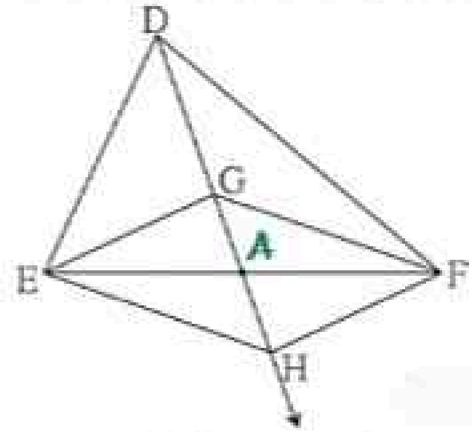
PROOF:

$$\left.\begin{aligned} \angle A &\cong \angle C = 90^\circ \\ \& \quad \angle B &\cong \angle D = 90^\circ \end{aligned}\right\} \quad \ldots \text{(Angles of a rectangle)}$$

$\therefore$ Rectangle ABCD is a parallelogram

$\ldots$ [A quadrilateral is a parallelogram if its opposite angles are congruent]

3. In figure 5.23, G is the point of concurrence of medians of Δ DEF. Take point H on ray DG such that D-G-H and DG = GH, then prove that □GEHF is a parallelogram.



GIVEN : G is a centroid of ΔDEF, DG = GH, D-G-H

TO PROVE : □GEHF is a parallelogram

PROOF :

Let median DG intersect side EF at 'A'

$\therefore$ AE = AF

Also, G divides DA in the ratio 2:1

Let the common multiple be x

$\therefore$ DG = $2x$ & GA = $1x$ ——— 1

Now, DG = GH = $2x$ ... (Given) ——— 2

Now, GH = GA + AH ... (G – A – H)

$\therefore$ $2x$ = $1x$ + AH ... (From 1 & 2)

$\therefore$ $2x - 1x$ = AH

$\therefore$ $1x$ = AH ——— 3

$\therefore$ AH = GA .. (From 1 & 3) ——— 4

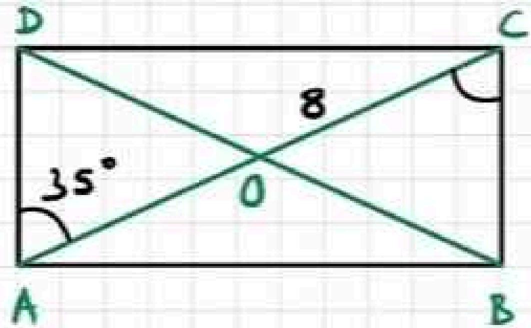
$\therefore$ Diagonals of □GEHF bisect each other

$\therefore$ □GEHF is a parallelogram

.. [A quadrilateral is a parallelogram if its diagonals bisect each other]

1. Diagonals of a rectangle ABCD intersect at point O. If AC = 8 cm then find the length of BO and if $\angle CAD = 35^\circ$ then find the measure of $\angle ACB$.

GIVEN: Diagonals of rectangle ABCD meet at O, AC = 8 cm, $\angle CAD = 35^\circ$



FIND: BO, $\angle ACB$

SOLUTION:

① $BD = AC = 8$ cm ... (Diagonals of rectangle are congruent)

Also, $BO = \frac{1}{2} BD$ ... (Diagonals of rectangle bisect each other)

$= \frac{1}{\cancel{2}_1} \times \cancel{8}^4$

$= 4$ cm $\quad \therefore \boxed{BO = 4\text{cm}}$

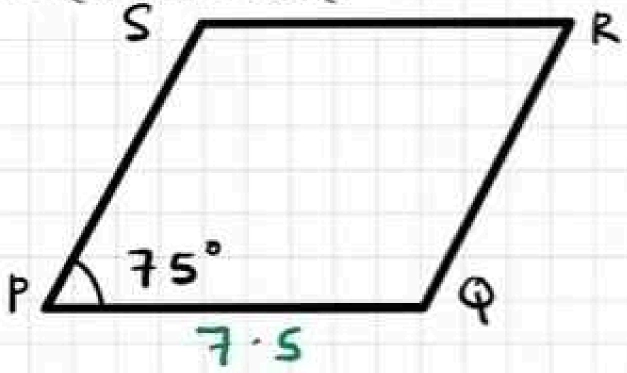
② Opposite sides of a rectangle are parallel

$\therefore$ Side $AD \parallel BC$ & AC is their transversal

$\therefore \angle ACB = \angle CAD$ ... (Alternate angles)

$\therefore \boxed{\angle ACB = 35^\circ}$ ... $(\because \angle CAD = 35^\circ)$

2. In a rhombus PQRS if PQ = 7.5 then find the length of QR.
If $\angle QPS = 75^\circ$ then find the measure of $\angle PQR$ and $\angle SRQ$.



GIVEN $PQ = 7 \cdot 5$

$\angle QPS = 75^\circ$

FIND: QR, $\angle PQR$ & $\angle SRQ$

SOLUTION:

① $QR = PQ$ ... (Sides of rhombus are congruent)

$\therefore QR = 7 \cdot 5$ ... ($\because PQ = 7 \cdot 5$)

$\therefore \boxed{QR = 7 \cdot 5 \text{ units}}$

② $\angle SRQ = \angle QPS$ ... (Opposite angles of Rhombus)

$\therefore \boxed{\angle SRQ = 75^\circ}$ ... ($\because \angle QPS = 75^\circ$)

③ $\angle PQR + \angle QPS = 180^\circ$ ... (Adjacent angles of a rhombus are supplementary)

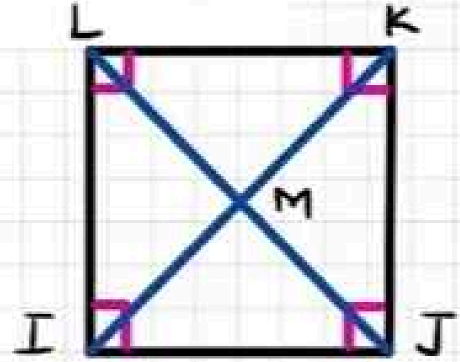
$\therefore \angle PQR + 75^\circ = 180^\circ$ ... ($\because \angle QPS = 75^\circ$)

$\therefore \angle PQR = 180^\circ - 75^\circ$

$\therefore \boxed{\angle PQR = 105^\circ}$

Practice set 5.3

3. Diagonals of a square IJKL intersects at point M, Find the measures of ∠IMJ, ∠JIK and ∠LJK .



GIVEN Diagonals of square intersect at M.

FIND : ∠IMJ, ∠JIK, ∠LJK

SOLUTION:

① Seg LJ ⊥ Seg IK ... (Diagonals of a square are perpendicular to each other)

$\therefore \boxed{\angle IMJ = 90^\circ}$

② $\angle JIK = \frac{1}{2} \angle JIL$ ... (Diagonals of a square bisect opposite angles)

$= \frac{1}{\cancel{2}_1} \times \cancel{90}^{45}{}^\circ$ ... ($\angle JIL = 90^\circ$, Angle of a square)

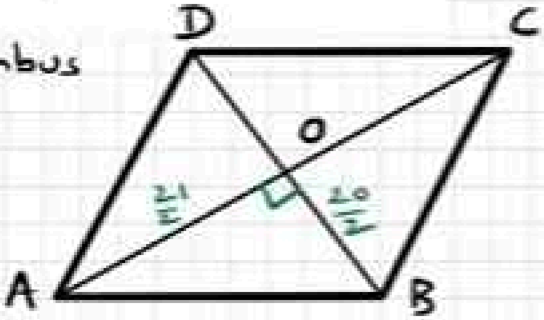
$\therefore \boxed{\angle JIK = 45^\circ}$

③ $\angle LJK = \frac{1}{2} \angle IJK$ ... (Diagonals of a square bisect opposite angles)

$= \frac{1}{2} \times 90^\circ$ ... ($\angle IJK = 90^\circ$, Angle of a square)

$\therefore \boxed{\angle LJK = 45^\circ}$

4. Diagonals of a rhombus are 20 cm and 21 cm respectively, then find the side of rhombus and its perimeter.



GIVEN Let $\square ABCD$ be a rhombus

$AC = 21$ cm, $BD = 20$ cm

FIND: Side and perimeter of rhombus

SOLUTION:

(1) Let diagonals intersect at O

$\therefore BO = \frac{1}{2} BD$ ... (Diagonals of Rhombus bisect each other)

$= \frac{1}{2} \times 20$

$= \frac{20}{2}$ cm —— (1)

Also, $AO = \frac{1}{2} AC$ ... (Diagonals of Rhombus bisect each other)

$= \frac{1}{2} \times 21$

$AO = \frac{21}{2}$ cm —— (2)

(2) $AC \perp BD$ ... (Diagonals of Rhombus are perpendicular to each other)

$\therefore \angle AOB = 90°$

n $\triangle AOB$, $AB^2 = AO^2 + BO^2$ ... (Pythagoras Theorem)

$= \left(\frac{21}{2}\right)^2 + \left(\frac{20}{2}\right)^2$ ... (From 1 & 2)

$= \frac{441}{4} + \frac{400}{4} = \frac{400+441}{4}$

$= \frac{841}{4}$

$\therefore AB = \sqrt{\frac{841}{4}} = \frac{29}{2} = 14.5$ cm

$\therefore \boxed{AB = 14.5 \text{ cm}}$

Practice set 5.3

5. State with reasons whether the following statements are ‘true’ or ‘false’.

(i) Every parallelogram is a rhombus. False

(ii) Every rhombus is a rectangle. False

(iii) Every rectangle is a parallelogram. True

(iv) Every squre is a rectangle. True

(v) Every square is a rhombus. True

(vi) Every parallelogram is a rectangle. False

Rhombus – All sides should be congruent

Rectangle – All angles should be 90°

Parallelogram – Opposite sides parallel

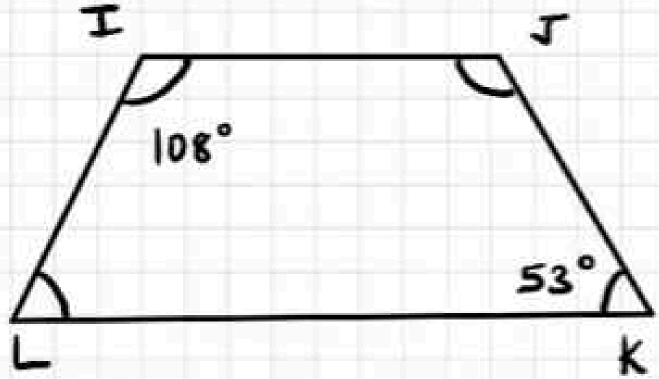
Square – All sides and angles are equal

1. In □IJKL, side IJ $\parallel$ side KL $\angle I = 108^\circ$ $\angle K = 53^\circ$ then find the measures of $\angle J$ and $\angle L$.

GIVEN : $IJ \parallel KL$

$\angle I = 108^\circ$, $\angle K = 53^\circ$

FIND : $\angle J$ & $\angle L$



SOLUTION:

1. $IJ \parallel KL$ and IL is their transversal

$\therefore \angle I + \angle L = 180^\circ$ ... (Interior Angles)

$\therefore 108^\circ + \angle L = 180^\circ$ ... ($\because \angle I = 108^\circ$, given)

$\therefore \angle L = 180^\circ - 108^\circ$

$\therefore \boxed{\angle L = 72^\circ}$

2. $IJ \parallel KL$ and JK is their transversal

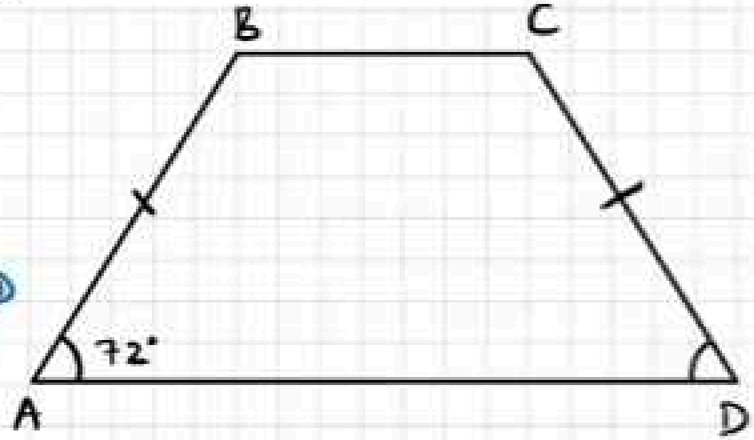
$\therefore \angle K + \angle J = 180^\circ$ ... (Interior Angles)

$\therefore 53^\circ + \angle J = 180^\circ$ ... ($\because \angle K = 53^\circ$, given)

$\therefore \angle J = 180^\circ - 53^\circ$

$\therefore \boxed{\angle J = 127^\circ}$

2. In $\square ABCD$, side $BC \parallel$ side AD, side $AB \cong$ side DC If $\angle A = 72^\circ$ then find the measures of $\angle B$, and $\angle D$.



GIVEN : $BC \parallel AD$,

$AB \cong DC$, $\angle A = 72^\circ$

FIND : $\angle B$, $\angle D$

CONSTRUCTION: Draw $BM \perp AD$ & $CN \perp AD$, A—M—N—D

SOLUTION:

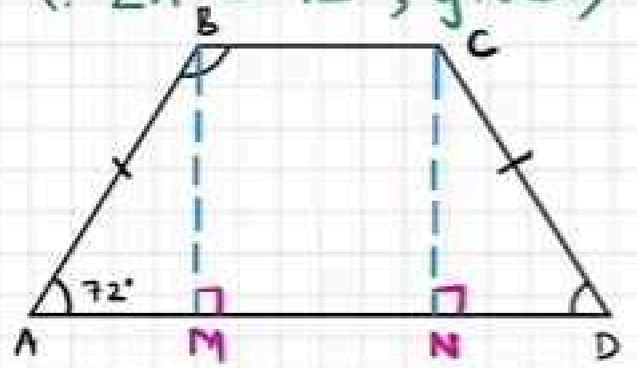
① $BC \parallel AD$ and AB is their transversal

$\therefore \angle A + \angle B = 180^\circ$ ... (Interior Angles)

$\therefore 72^\circ + \angle B = 180^\circ$ ... ($\because \angle A = 72^\circ$, given)

$\therefore \angle B = 180^\circ - 72^\circ$

$\therefore \boxed{\angle B = 108^\circ}$



② In right angled triangles $\triangle BMA$ & $\triangle CND$

$\angle BMA \cong \angle CND$ ... (Each angle is 90°)

Hypotenuse $BA \cong$ Hypotenuse CD ... (Given)

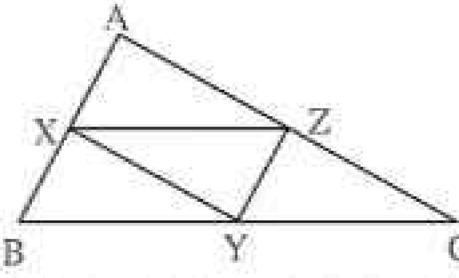
Side $BM \cong$ Side CN ... (Distance between parallel lines)

$\therefore \triangle BMA \cong \triangle CND$ ... (Hypotenuse Side Test)

$\therefore \angle A \cong \angle D$ ... (c.a.c.t)

$\therefore \boxed{\angle D = 72^\circ}$ ... ($\because \angle A = 72^\circ$, given)

1. In figure 5.38, points X, Y, Z are the midpoints of side AB, side BC and side AC of Δ ABC respectively. AB = 5 cm, AC = 9 cm and BC = 11 cm. Find the length of XY, YZ, XZ.



GIVEN: X, Y, Z are midpoints of side AB, BC & AC. AB = 5cm, AC = 9cm, BC = 11cm

FIND: XY, YZ, XZ

SOLUTION:

i) Points X & Y are midpoints of sides AB & BC ... (Given)

$\therefore \quad XY = \frac{1}{2} AC \quad$ ... (Midpoint theorem)

$= \frac{1}{2} \times 9 \quad$ ... ($\because$ AC = 9cm, given)

$= 4.5$ cm

ii) Points Y & Z are midpoints of sides BC & AC ... (Given)

$\therefore \quad YZ = \frac{1}{2} BA \quad$ ... (Midpoint theorem)

$= \frac{1}{2} \times 5 \quad$ ... ($\because$ BA = 5cm, given)

$= 2.5$ cm

iii) Points X & Z are midpoints of sides AB & AC ... (Given)

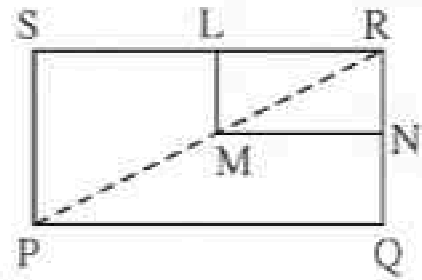
$\therefore \quad XZ = \frac{1}{2} BC \quad$ ... (Midpoint theorem)

$= \frac{1}{2} \times 11 \quad$ ... ($\because$ BC = 11cm, given)

$= 5.5$ cm

$\therefore \quad \boxed{XY = 4.5 \text{ cm}, \quad YZ = 2.5 \text{ cm}, \quad XZ = 5.5 \text{ cm}}$

2. In figure 5.39, $\square$PQRS and $\square$MNRL are rectangles. If point M is the midpoint of side PR then prove that, (i) SL = LR, (ii) LN = $\frac{1}{2}$SQ.



GIVEN: $\square$PQRS & $\square$MNRL are rectangles, M is the midpoint of PR

TO PROVE: (i) SL = LR (ii) LN = $\frac{1}{2}$ SQ

PROOF:

(i) $\angle S = \angle L = 90°$ ... (Angles of rectangle $\square$PQRS & $\square$MNRL)

But,

$\angle S$ & $\angle L$ are corresponding angles on sides SP & LM with RS as their transverse

$\therefore$ SP ∥ LM —① ... (Corresponding Angles Test)

Now, In $\triangle$PRS,

M is the midpoint of PR ... (given)

& SP ∥ LM ... (from 1)

$\therefore$ Point L is the midpoint of RS —② ... (Converse of midpoint theorem)

$\therefore$ $\boxed{SL = LR}$

ii) Similarly in $\triangle$RPQ,

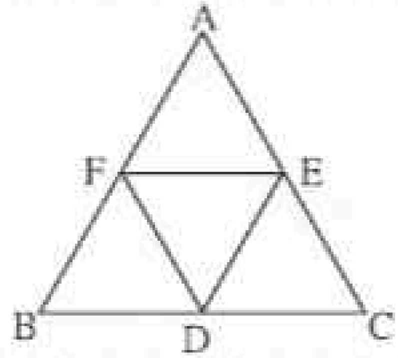
N is the midpoint of RQ —③

Now, in $\triangle$RSQ

L & N are midpoints of RS & RQ ... (from 2 & 3)

$\therefore$ $\boxed{LN = \frac{1}{2} SQ}$ ... (Midpoint theorem)

3. In figure 5.40, ΔABC is an equilateral traingle. Points F,D and E are midpoints of side AB, side BC, side AC respectively. Show that Δ FED is an equilateral traingle.



GIVEN: ΔABC is an equilateral triangle. F, D, E are midpoints of AB, BC and AC respectively

TO PROVE: ΔFED is an equilateral triangle.

PROOF:

Points F & E are midpoints of sides AB & AC ... (Given)

$\therefore$ $FE = \frac{1}{2} BC$ — ① ... (Midpoint theorem)

Points F & D are midpoints of sides AB & BC ... (Given)

$\therefore$ $FD = \frac{1}{2} AC$ — ② ... (Midpoint theorem)

Points D & E are midpoints of sides BC & AC ... (Given)

$\therefore$ $DE = \frac{1}{2} AB$ — ③ ... (Midpoint theorem)

But, $AB = BC = AC$ — ④ ... (ΔABC is an equilateral triangle)

$\therefore$ $FE = FD = DE$ ... (From 1, 2, 3 & 4)

$\therefore$ ΔFED is an equilateral triangle.