REAL NUMBERS

Revision Notes on Real Numbers

Euclid's Division Lemma

It is basically the restatement of the usual division system. The formal statement for this is-

For each pair of given positive integers *a* and *b*, there exist unique whole numbers *q* and *r* which satisfies the relation

$\mathbf{a = bq + r, 0 \le r < b}$, where q and r can also be Zero.

where 'a' is a dividend, 'b' is divisor, 'q' is quotient and 'r' is remainder.

∴ Dividend = (Divisor x Quotient) + Remainder

Natural Numbers

Non-negative counting numbers excluding zero are known as natural numbers.

i.e. 5, 6, 7, 8, ...

Whole numbers

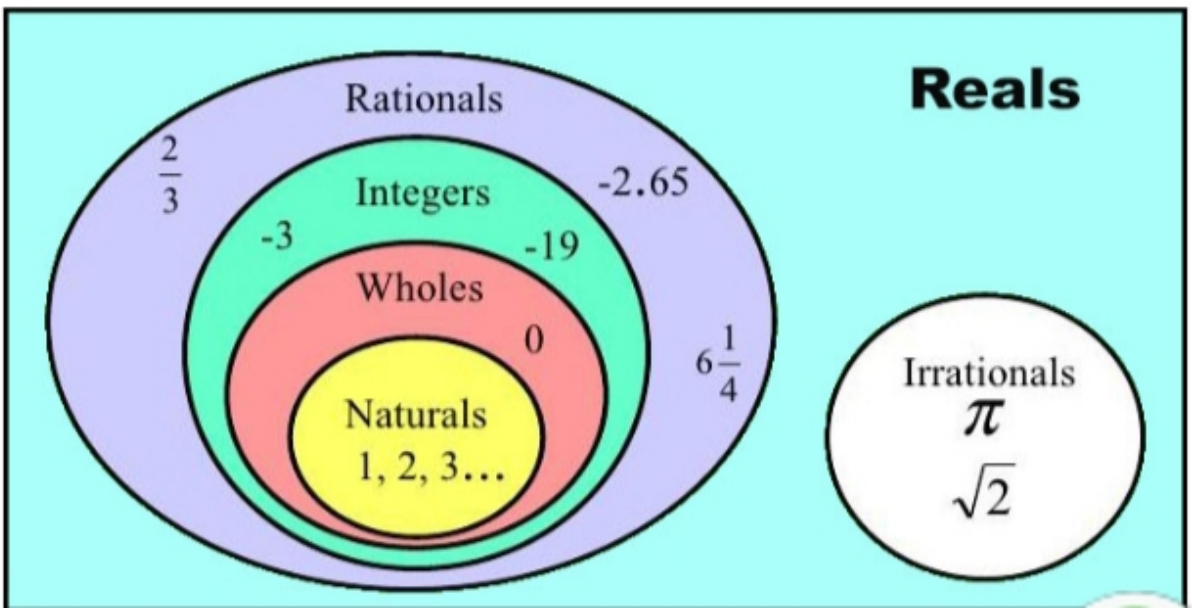
All non-negative counting numbers including zero are known as whole numbers.

i.e. 0, 1, 2, 3, 4, 5, ...

Integers

All negative and non-negative numbers including zero altogether known as integers.

i.e. .... – 3, – 2, – 1, 0, 1, 2, 3, 4, ...



REAL NUMBERS DEFINITION

Rational $\frac{5}{3}$ 0.63 $0.0\overline{12}$

Integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Whole $\{0, 1, 2, 3, \ldots\}$

Natural $\{1, 2, 3, \ldots\}$

Irrational

$\sqrt{3}$ π $0.10100110\ldots$

Real Numbers Definition

Real numbers can be defined as the union of both the rational and irrational numbers. They can be both positive or negative and are denoted by the symbol "R". All the natural numbers, decimals and fractions come under this category. See the figure, given below, which shows the classification of real numerals.

Step 1: Apply Euclid's division lemma to find q and r where $m = nq + r$, $0 \leq r < n$.

Step 2: If the remainder i.e. $r = 0$, then the HCF will be 'n' but if $r \neq 0$ then we have to apply Euclid's division lemma to n and r.

Step 3: Continue with this process until we get the remainder as zero. Now the divisor at this stage will be HCF(m, n). Also, HCF (m, n) = HCF (n, r), where HCF (m, n) means HCF of m and n.

Algorithm

An algorithm gives us some definite steps to solve a particular type of problem in a well-defined manner.

Lemma

A lemma is a statement which is already proved and is used for proving other statements.

Euclid's Division Algorithm

This concept is based on Euclid's division lemma. This is the technique to calculate the HCF (Highest common factor) of given two positive integers m and n,

To calculate the HCF of two positive integers' m and n with m > n, the following steps are followed:

Ex 1.1 , 1

Use Euclid's division algorithm to find the HCF of :

(i) 135 and 225

Since 225 > 135,

We divide 225 by 135

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135 ) 225 ( 1
  (-) 135
      ---
       90
      ---
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Since remainder is not 0

We divide 135 by 90

$$90 \overline{)\,135\,}\; 1$$

$$\begin{array}{r} (-)\;\;90 \\ \hline 45 \\ \hline \end{array}$$

Again, since remainder is not 0

We divide 90 by 45

$$45 \overline{)\,90\,}\; 2$$

$$\begin{array}{r} (-)\;\;90 \\ \hline 0 \\ \hline \end{array}$$

Since remainder is now 0

HCF of 135 and 225 is 45

Ex 1.1 , 2

Show that any positive odd integer is of the form 6q + 1, or 6q+ 3, or 6q+ 5, where q is some integer.

As per Euclid's Division Lemma

If a and b are 2 positive integers, then

$a = bq + r$

where $0 \leq r < b$

Let positive integer be a

And $b = 6$

Hence $a = 6q + r$

where $(0 \leq r < 6)$

r is an integer greater than or equal to 0 and less than 6

hence r can be either 0 , 1 , 2 ,3 , 4 or 5

If r = 1

Our equation becomes

$a = 6q + r$

$a = 6q + 1$

This will always be an odd integer

If r = 3

Our equation becomes

$a = 6q + r$

$a = 6q + 3$

This will always be an odd integer

If r = 5

Our equation becomes

$a = 6q + r$

$a = 6q + 5$

This will always be an odd integer

Therefore, any odd integer is of the form $6q + 1$, $6q + 3$ or $6q + 5$

Hence proved

H.W

(iii)867 and 255