Theorem 6.1

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Sol^n] Proof:

We are given a triangle ABC in which a line parallel to side ~~BC~~ AB & AC at D & E respectively

We need to prove that

$\frac{AD}{DB} = \frac{AE}{EC}$

Const^n] Let us join BE & CD & then draw $DM \perp AC$ & $EN \perp AB$.

Now, area of $\Delta ADE (= \frac{1}{2} \times b \times h) = \frac{1}{2} \times AD \times EN$

Recall that area of ΔADE is denoted ~~as~~ as ar(ADE)

So, $ar(ADE) = \frac{1}{2} AD \times EN$

$ar(ADE) = 1/2 \times AE \times DM$ & $ar(DEC) = 1/2\, EC \times DM$.

$\therefore \frac{ar(ADE)}{ar(BDE)} \frac{1/2 \times AD \times EN}{1/2 \times BD \times EN} = \frac{AD}{DB}$

& $\frac{ar(ADE)}{ar(DEC)} \frac{1/2\, AE \times DM}{1/2\, EC \times DM} = \frac{\cancel{ADE}}{\cancel{DB}\,EC} \frac{AE}{EC}$

Now, ΔBDE & ΔDEC are on same base DE & b/w the same parallels BC & DE.

Also, ar(BDE) = ar(DEC)

$\therefore$ From (i), (ii) & (iii)

$$\frac{AD}{DB} = \frac{AE}{EC}$$ //

~~Is the converse of~~