Objective

To establish a formula for the sum of first n terms of an Arithmetic Progression.

Material Required

Cardboard, coloured drawing sheets, white paper, cutter, adhesive.

Method of Construction

1. Take a rectangular cardboard of a convenient size and paste a white paper on it. Draw a rectangle ABCD of length $(2a+9d)$ units and breadth 10 units.
2. Make some rectangular strips of equal length a units and breadth one unit and some strips of length d units and breadth 1 unit, using coloured drawing sheets.
3. Arrange/paste these strips on the rectangle ABCD as shown in Fig. 1.

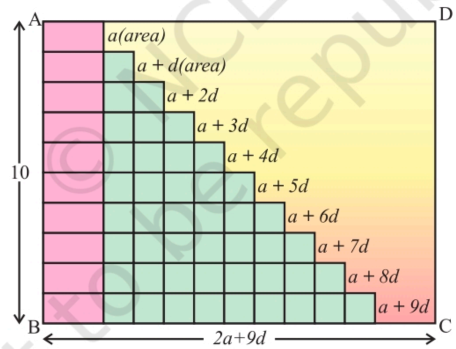

Fig. 1

Demonstration

1. The strips so arranged look like a stair case.
2. The first stair is of length a units, the second stair is of length $a+d$ (units), third of $a+2d$ units and so on and each is of breadth 1 unit. So, the areas (in sq. units) of these strips are a, $a + d$, $a + 2d$, ..., $a+9d$, respectively.

3. This arrangement of strips gives a pattern $a, a + d, a + 2d, a + 3d, ...$ which is an AP with first term a and the common difference d.
4. The sum of the areas (in square units) of these strips
   $= a + (a + d) + (a + 2d) + ... + (a + 9d) = 10a + 45d$ (1)
5. Area of the designed formed by the stair case $= \frac{1}{2}$ (area of rectangle ABCD)

$= \frac{1}{2}(10)(2a+9d)$

$= (10a + 45d)$, which is the same as obtained in (1) above.

This shows that the sum of first 10 terms of the AP $= \frac{1}{2}(10)(2a+9d)$

$= \frac{1}{2}(10)\,[2a+(10-1)d]$

This can be further generalised to find the sum of first n terms of an AP as

$S_n = \frac{n}{2}\,[2a+(n-1)d]$

Observation

On actual measurement:

a = ---------, d = -----------, n = --------- S_n = ------------

So, $S_n = \frac{n}{2}\,[-+(n-1)-]$.

Objective

To find the sum of first n natural numbers.

Material Required

Cardboard, coloured papers, white paper, cutter, adhesive.

Method of Construction

1. Take a rectangular cardboard of a convenient size and paste a coloured paper on it. Draw a rectangle ABCD of length 11 units and breadth 10 units.
2. Divide this rectangle into unit squares as shown in Fig. 1.
3. Starting from upper left-most corner, colour one square, 2 squares and so on as shown in the figure.

Demonstration

1. The pink colour region looks like a stair case.
2. Length of 1st stair is 1 unit, length of 2nd stair is 2 units, length of 3rd stair 3 units, and so on, length of 10th stair is 10 units.

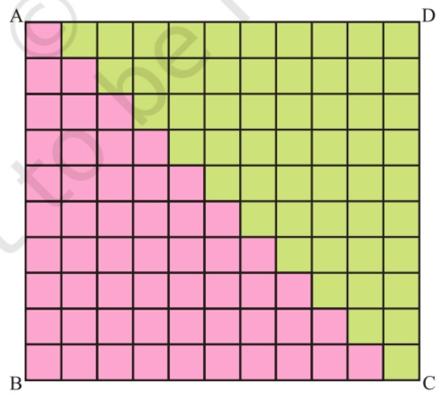

Fig. 1

3. These lengths give a pattern

   1, 2, 3, 4, ..., 10,

   which is an AP with first term 1 and common difference 1.

4. Sum of first ten terms

   $= 1 + 2 + 3 + ... + 10 = 55$ (1)

   Area of the shaded region $= \frac{1}{2}$ (area of rectangle ABCD)

   $= \frac{1}{2} \times 10 \times 11$, which is same as obtained in (1) above. This shows that the sum of the first 10 natural numbers is $\frac{1}{2} \times 10 \times 11 = \frac{1}{2} \times 10(10+1)$.

   This can be generalised to find the sum of first n natural numbers as

   $$S_n = \frac{1}{2} n(n+1) \quad (2)$$

Observation

For $n = 4$, S_n = ...

For $n = 12$, S_n = ...

For $n = 50$, S_n = ...

For $n = 100$, S_n = ...

OBJECTIVE

To find the sum of the first n odd natural numbers.

MATERIAL REQUIRED

Cardboard, thermocol balls, pins, pencil, ruler, adhesive, white paper.

METHOD OF CONSTRUCTION

1. Take a piece of cardboard of a convenient size and paste a white paper on it.
2. Draw a square of suitable size on it (10 cm × 10 cm).
3. Divide this square into unit squares.
4. Fix a thermocol ball in each square with the help of a pin as shown in Fig. 1.
5. Enclose the balls as shown in the figure.

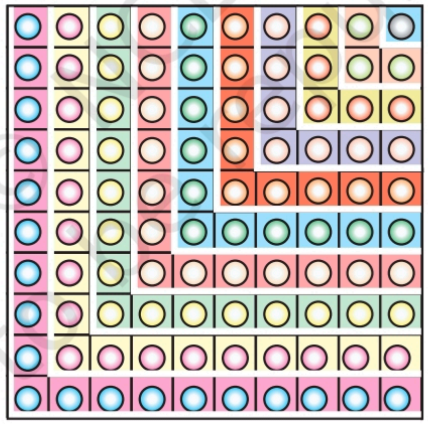

Fig. 1

DEMONSTRATION

Starting from the uppermost right corner, the number of balls in first enclosure (blue colour) = 1 $(=1^2)$,

the number of balls in first 2 enclosures = 1 + 3 = 4 $(=2^2)$,

the number of balls in first 3 enclosures = 1 + 3 + 5 = 9 $(=3^2)$,

the number of balls in first 10 enclosures = 1 + 3 + 5 + ... + 19 = 100 $(=10^2)$.

This gives the sum of first ten odd natural numbers. This result can be generalised for the sum of first n odd numbers as:

$S_n = 1 + 3 + \ldots\ldots\ldots + (2n - 1) = n^2$ (1)

OBSERVATION

For $n = 4$ in (1), S_n =.......................

For $n = 5$ in (1), S_n =.......................

For $n = 50$ in (1), S_n =.......................

For $n = 100$ in (1), S_n= ....

APPLICATION

The activity is useful in determining formula for the sum of the first n odd natural numbers.

Objective

To find the sum of the first n-even natural numbers.

Material Required

Cardboard, thermocol balls, pins, pencil, ruler, white paper, chart paper, adhesive.

Method of Construction

1. Take a piece of cardboard of a convenient size and paste a white paper on it.
2. Draw a rectangle of suitable size on it (10 cm × 11 cm).
3. Divide this rectangle into unit squares.
4. Fix a thermocol ball in each square using a pin as shown in the Fig. 1.
5. Enclose the balls as shown in the figure.

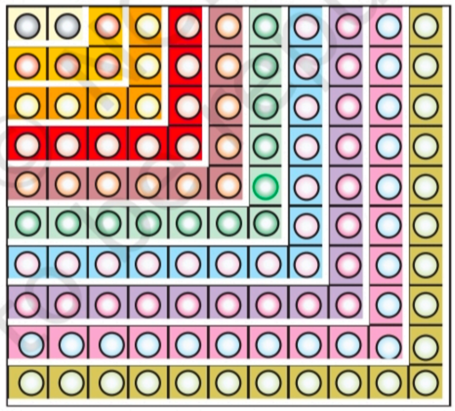

Fig. 1

Demonstration

Starting from the uppermost left corner,

the number of balls in first enclosure = 2 (= 1 ×2),

the number of balls in first two enclosures = 2 + 4 = 6 (= 2 ×3),

the number of balls in first three enclosures = 2 + 4 + 6 = 12 (= 3 ×4),

⋮

the number of balls in first six enclosures = 2 + 4 + 6 + 8 + 10 + 12 = 42 (= 6 × 7)

the number of balls in first ten enclosures = 2 +4 + 6 + 8 + ... + 20 = 110 (=10 × 11)

This gives the sum of first ten even natural numbers.

This result can be generalised for the sum of first n even natural numbers as

$S_n = 2 + 4 + 6 + ... + 2n = n \times (n + 1)$ (1)

Observation

For $n = 4$ in (1), S_n = ...

For $n = 7$ in (1), S_n = ...

For $n = 40$ in (1), S_n = ...

For $n = 70$ in (1), S_n = ...

For $n = 100$ in (1), S_n = ...